Module 4: DC-DC Converters

Lec 9: DC-DC Converters for EV and HEV Applications

DC-DC Converters for EV and HEV Applications

Introduction

The topics covered in this chapter are as follows:

- EV and HEV configuration based on power converters
- Classification of converters
- Principle of Step Down Operation
- Buck Converter with RLE Load
- Buck Converter with RL Load and Filter

Electric Vehicle (EV) and Hybrid Electric Vehicle (HEV) Configurations

In Figure 1 the general configuration of the EV and HEV is shown. Upon examination of the general configurations it can be seen that there are two major power electronic units

- DC-DC converter
- DC-AC inverter

![Figure 1: General Configuration of a Electric Vehicle](image)
Usually AC motors are used in HEVs or EVs for traction and they are fed by inverter and this inverter is fed by DC-DC converter (Figure 1). The most commonly DC-DC converters used in an HEV or an EV are:

- **Unidirectional Converters**: They cater to various onboard loads such as sensors, controls, entertainment, utility and safety equipments.

- **Bidirectional Converters**: They are used in places where battery charging and regenerative braking is required. The power flow in a bi-directional converter is usually from a low voltage end such as battery or a supercapacitor to a high voltage side and is referred to as *boost operation*. During regenerative braking, the power flows back to the low voltage bus to recharge the batteries know as *buck mode* operation.

Both the unidirectional and bi-directional DC-DC converters are preferred to be *isolated* to provide safety for the lading devices. In this view, most of the DC-DC converters incorporate a high frequency transformer.

**Classification of Converters**

The converter topologies are classified as:

- **Buck Converter**: In Figure 2a a buck converter is shown. The buck converter is *step down* converter and produces a lower average output voltage than the dc input voltage.

- **Boost converter**: In Figure 2b a boost converter is shown. The output voltage is always greater than the input voltage.

- **Buck-Boost converter**: In Figure 2c a buck-boost converter is shown. The output voltage can be either higher or lower than the input voltage.
Figure 2a: General Configuration Buck Converter

Figure 2b: General Configuration Boost Converter

Figure 2c: General Configuration Buck-Boost Converter
Principle of Step Down Operation

The principle of step down operation of DC-DC converter is explained using the circuit shown in Figure 3a. When the switch $S_1$ is closed for time duration $T_1$, the input voltage $V_{in}$ appears across the load. For the time duration $T_2$ is switch $S_1$ remains open and the voltage across the load is zero. The waveforms of the output voltage across the load are shown in Figure 3b.

The average output voltage is given by

$$V_{avg} = \frac{1}{T} \int_0^T v_{out} \, dt = \frac{T_1}{T} V_{in} = fT_1 V_{in} = DV_{in}$$ \hspace{1cm} (1)

The average load current is given by

$$I_{avg} = \frac{V_{avg}}{R} = \frac{DV_{in}}{R}$$ \hspace{1cm} (2)

Where

$T$ is the chopping period

$D = \frac{T_1}{T}$ is the duty cycle

$f$ is the chopping frequency

The rms value of the output voltage is given by

$$V_{rms} = \left( \frac{1}{T} \int_0^T v_{out}^2 \, dt \right)^{1/2} = \sqrt{DV_{in}}$$ \hspace{1cm} (3)
In case the converter is assumed to be lossless, the input power to the converter will be equal to the output power. Hence, the input power \( P_{\text{in}} \) is given by

\[
P_{\text{in}} = \frac{1}{T} \int_0^{DT} v_{\text{out}} i_{\text{out}} dt = \frac{1}{T} \int_0^{DT} \frac{V_{\text{out}}^2}{R} dt = D \frac{V_{\text{in}}^2}{R}
\]

The effective resistance seen by the source is (using equation 2)

\[
R_{\text{eff}} = \frac{V_{\text{in}}}{I_{\text{avg}}} = \frac{R}{D}
\]

The duty cycle \( D \) can be varied from 0 to 1 by varying \( T_1, T \) or \( f \). Thus, the output voltage \( V_{\text{avg}} \) can be varied from 0 to \( V_{\text{in}} \) by controlling \( D \) and eventually the power flow can be controlled.

The Buck Converter with RLE Load

The buck converter is a voltage step down and current step up converter. The two modes in steady state operations are:

**Mode 1 Operation**

In this mode the switch \( S_1 \) is turned on and the diode \( D_1 \) is reversed biased, the current flows through the load. The time domain circuit is shown in Figure. The load current, in \( s \) domain, for mode 1 can be found from

\[
R_i(s) + sL_i(s) + \frac{E}{s} = \frac{V_{\text{in}}}{s} + L_i01
\]

Where

\( I_{01} \) is the initial value of the current and \( I_{01} = I_1 \).
From equation 6, the current \( i_1(s) \) is given by

\[
i_1(s) = \frac{(V_{in} - E)}{s(R + sL)} + \frac{LI_1}{R + sL}
\]  

In time domain the solution of equation 7 is given by

\[
i_1(t) = I_1 e^{-\frac{R}{L}t} + \frac{V_{in} - E}{R}(1 - e^{-\frac{R}{L}t})
\]  

The **mode 1** is valid for the time duration \( 0 \leq t \leq T_1 \Rightarrow 0 \leq t \leq DT \). At the end of this mode, the load current becomes

\[
i_1(t = T_1 = DT) = I_2
\]

**Mode 2 Operation**

In this mode the switch \( S_1 \) is turned off and the diode \( D_1 \) is forward biased. The time domain circuit is shown in Figure 5. The load current, in \( s \) domain, can be found from

\[
Ri_2(s) + sLi_2(s) + \frac{E}{s} = LI_{02}
\]

Where

\( I_{02} \) is the initial value of load current.

The current at the end of **mode 1** is equal to the current at the beginning of **mode 2**. Hence, from equation 9 \( I_{01} \) is obtained as

\[
I_{01} = I_2
\]

Hence, the load current is time domain is obtained from equation 10 as

\[
i_2(t) = I_2 e^{-\frac{R}{L}t} - \frac{E}{R}(1 - e^{-\frac{R}{L}t})
\]

**Determination of \( I_1 \) and \( I_2 \)**

At the end of **mode 2** the load current becomes

\[
i_2(t = T_2 = (1 - D)T) = I_3
\]

At the end of **mode 2**, the converter enters **mode 1** again. Hence, the initial value of current in **mode 1** is

\[
I_{01} = I_3 = I_1
\]

From equation 8 and equation 12 the following relation between \( I_1 \) and \( I_2 \) is obtained as

\[
I_2 = I_1 e^{-\frac{DTR}{L}} + \frac{V_{in} - E}{R}(1 - e^{-\frac{DTR}{L}})
\]

\[
I_3 = I_1 e^{-(1-D)\frac{TR}{L}} - \frac{E}{R}(1 - e^{-(1-D)\frac{TR}{L}})
\]
Solving equation 15 and equation 16 for $I_1$ and $I_2$ gives

$$I_1 = \frac{V_m}{R} \left( e^{D_a} - 1 \right) - \frac{E}{R}$$

(17)

$$I_2 = \frac{V_m}{R} \left( e^{-D_a} - 1 \right) - \frac{E}{R}$$

(18)

Where

$$a = \frac{TR}{L} = \frac{R}{f_L}$$

(19)

where $f$ is the chopping frequency.

**Current Ripple**

The peak to peak current ripple is given by

$$\Delta I = I_2 - I_1 = \frac{V_m}{R} \frac{1 - e^{-D_a} + e^{-a} - e^{-(1-D)a}}{1 - e^{-a}} = \frac{V_m}{f L} \frac{1 - e^{-D_a} + e^{-a} - e^{-(1-D)a}}{a \left(1 - e^{-a}\right)}$$

(20a)

In case $fL >> R$, $a \to 0$. Hence, for the limit $a \to 0$ equation 20 becomes

$$\Delta I = \frac{V_m D(1-D)}{fL}$$

(20b)

To determine the maximum current ripple ($\Delta I_{\text{max}}$), the equation 20a is differentiated w.r.t. $D$. The value of $\Delta I_{\text{max}}$ is given by

$$\Delta I_{\text{max}} = \frac{V_m}{R} \tanh \frac{R}{4fL}$$

(21)

For the condition $4fL >> R$,

$$\tanh \left( \frac{R}{4fL} \right) \approx \frac{R}{4fL}$$

(22)

Hence, the maximum current ripple is given by

$$\Delta I_{\text{max}} = \frac{V_m}{4fL}$$

(23)

If equation 20b is used to determine the maximum current ripple, the same result is obtained.
Continuous and Discontinuous Conduction Modes

In case of large off time, particularly at low switching frequencies, the load current may be discontinuous, i.e. \( i_2(t = T_2 = (1 - D)T) \) will be zero. The necessary condition to ensure continuous conduction is given by

\[
I_1 > 0 \Rightarrow \frac{V_{in}}{R} \left( \frac{e^{D_s} - 1}{e^{D_s} - 1} \right) - \frac{E}{R} \geq 0
\]

\[
\Rightarrow \frac{E}{V_{in}} \leq \left( \frac{e^{D_s} - 1}{e^{D_s} - 1} \right)
\]

(24)

The Buck Converter with R Load and Filter

The output voltage and current of the converter contain harmonics due to the switching action. In order to remove the harmonics LC filters are used. The circuit diagram of the buck converter with LC filter is shown in Figure 6. There are two modes of operation as explained in the previous section.

The voltage drop across the inductor in mode 1 is

\[
e_{L_j} = V_{in} - V_o = L_j \frac{di}{dt} \quad \text{and} \quad i_L = i_{sw}
\]

(25)

where \( i_L \) is the current through the inductor \( L_j \) and \( i_{sw} \) is the current through the switch.

The switching frequency of the converter is very high and hence, \( i_L \) changes linearly.

Thus, equation 25 can be written as

\[
e_{L_j} = V_{in} - V_o = L_j \frac{\Delta i_L}{T_{on}} = L_j \frac{\Delta i_{sw}}{DT}
\]

(26)

where \( T_{on} \) is the duration for which the switch \( S \) remains on and \( T \) is the switching time period.
Hence, the current ripple $\Delta i_L$ is given by

$$\Delta i_L = \frac{(V_m - V_o) DT}{L_f}$$

(27)

When the switch $S$ is turned off, the current through the filter inductor decreases and the current through the switch $S$ is zero. The voltage equation is

$$V_o = L_f \frac{di_L}{dt} = L_f \frac{di_D}{dt}$$

(28)

where $i_D$ is the current through the diode $D$

Due to high switching frequency, the equation 28 can be written as

$$V_o = L_f \frac{\Delta i_L}{T_{off}} = L_f \frac{\Delta i_L}{(1 - D)T}$$

(29)

where $T_{off}$ is the duration in which switch $S$ remains off the diode $D$ conducts

Neglecting the very small current in the capacitor $C_f$, it can be seen that $i_o = i_{sw}$ for time duration in which switch $S$ conducts

and

$i_o = i_D$ for the time duration in which the diode $D$ conducts

The current ripple obtained from equation 29 is

$$\Delta i_L = \frac{(1 - D)T}{L} V_o$$

(30)

The voltage and current waveforms are shown in Figure 7.
From equation 27 and equation 30 the following relation is obtained for the current ripple
\[
\Delta i_L = \frac{(V_m - V_o) DT}{L_f} = \frac{(1 - D) T}{L_f} V_o
\]  
(31)

Hence, from equation 31 the relation between input and output voltage is obtained as
\[
V_o = D V_m \Rightarrow \frac{V_o}{V_m} = D
\]  
(32)

If the converter is assumed to be lossless, then
\[
P_{in} = P_o \Rightarrow V_{in} i_{in} = V_o i_o \Rightarrow V_{in} i_{in} = D V_o i_o \Rightarrow i_{in} = D i_o
\]  
(33)

The switching period \( T \) can be expressed as
\[
T = \frac{1}{f} = T_{on} + T_{off} = L_f \frac{\Delta i_L}{V_{in} - V_o} + L_f \frac{\Delta i_L}{V_o} = L_f \frac{V_o \Delta i_L}{V_o (V_{in} - V_o)}
\]  
(34)

From equation 34 the current ripple is given by
\[
\Delta i_L = \frac{V_o (V_{in} - V_o)}{L_f V_o f}
\]  
(35)

Substituting the value of \( V_o \) from equation 32 into equation 35 gives
\[
\Delta i_L = \frac{V_m D (1 - D_o)}{f L_f}
\]  
(36)

Using the Kirchhoff’s current law, the inductor current \( i_L \) is expressed as
\[
i_L = i_c + i_o
\]  
(37)

If the ripple in load current (\( i_o \)) is assumed to be small and negligible, then
\[
\Delta i_L = \Delta i_c
\]  
(38)

The incremental voltage \( \Delta V_c \) across the capacitor (\( C_f \)) is associated with incremental charge \( \Delta Q \) by the relation
\[
\Delta V_c = \frac{\Delta Q_f}{C_f}
\]  
(39)

The area of each of the isosceles triangles representing \( \Delta Q \) in Figure 7 is given by
\[
\Delta Q_f = \frac{1}{2} \frac{T}{2} \frac{\Delta i_L}{2} = \frac{T \Delta i_L}{8}
\]  
(40)

Combining equation 39 and equation 40 gives
\[
\Delta V_c = \frac{T \Delta i_L}{8 C_f}
\]  
(41)
Substituting the value of $\Delta i_L$ from equation 31 into equation 41 gives

$$\Delta V_c = \frac{T}{8C_f} \frac{V_{in}D(1-D)}{fL_f} = \frac{V_{in}D(1-D)}{8L_fC_f\frac{f^2}{2}}$$

(42)

**Boundary between Continuous and Discontinuous Conduction**

The inductor ($i_L$) and the voltage drop across the inductor ($e_L$) are shown in Figure 8.

![Figure 8: The inductor voltage and current waveforms for discontinuous operation](image)

Figure 8: The inductor voltage and current waveforms for discontinuous operation

Figure 9: Current versus duty ratio keeping input voltage constant.

Being at the boundary between the continuous and the discontinuous mode, the inductor current $i_L$ goes to zero at the end of the off period. At this boundary, the average inductor current is (B refers to the boundary)

$$I_{LB} = \frac{1}{2} i_{L, peak} = \frac{T_{on}}{2L_f} (V_{in} - V_o) = \frac{DT}{2L_f} (V_{in} - V_o) = I_{oB}$$

(43)

Hence, during an operating condition, if the average output current ($I_L$) becomes less than $I_{LB}$, then $I_L$ will become discontinuous.

**Discontinuous Conduction Mode with Constant Input Voltage $V_{in}$**

In applications such as speed control of DC motors, the input voltage ($V_{in}$) remains constant and the output voltage ($V_o$) is controlled by varying the duty ratio $D$. Since $V_o = DV_{in}$, the average inductor current at the edge of continuous conduction mode is obtained from equation 43 as

$$I_{LB} = \frac{TV_{in}}{2L_f} D(1-D)$$

(44)
In Figure 9 the plot of $I_{lb}$ as a function of $D$, keeping all other parameters constant, is shown. The output current required for a continuous conduction mode is maximum at $D=0.5$ and by substituting this value of duty ratio in equation 44 the maximum current ($I_{lb,\text{max}}$) is obtained as

$$I_{lb,\text{max}} = \frac{TV_{in}}{8L} \quad (45)$$

From equation 44 and equation 45, the relation between $I_{lb}$ and $I_{lb,\text{max}}$ is obtained as

$$I_{lb} = 4I_{lb,\text{max}}D(1-D) \quad (46)$$

To understand the ratio of output voltage to input voltage ($V_o/V_{in}$) in the discontinuous mode, it is assumed that initially the converter is operating at the edge of the continuous conduction (Figure 7), for given values of $T, L, V_d$ and $D$. Keeping these parameters constant, if the load power is decreased (i.e., the load resistance is increased), then the average inductor current will decrease. As is shown in Figure 10, this dictates a higher value of $V_o$ than before and results in a discontinuous inductor current.

![Figure 10: Discontinuous operation is buck converter](image)

![Figure 11: Buck converter characteristics for constant input current](image)

In the time interval $\Delta T$ the current in the inductor $L_f$ is zero and the power to the load resistance is supplied by the filter capacitor alone. The inductor voltage $e_L$ during this time interval is zero. The integral of the inductor voltage over one time period is zero and in this case is given by

$$\left(V_{in} - V_o\right)DT + (-V_o)\Delta T_s = 0 \Rightarrow \frac{V_o}{V_{in}} = \frac{D}{D+\Delta} \quad (47)$$
In the interval $0 \leq t \leq \Delta t_s$ (Figure 10) the current ripple in $L_f$ is

$$e_L = L_f \frac{di_L}{dt} \Rightarrow e_L = L_f \frac{\Delta i_L}{\Delta t_s T}$$  \hspace{1cm} (48)

From Figure 10 it can be seen that

$$\Delta i_L = -i_{L,peak}$$  \hspace{1cm} (49)

$$e_L = -V_o$$  \hspace{1cm} (50)

Substituting the values of $\Delta i_L$ and $e_L$ from equation 49 and equation 50 into equation 48 gives

$$V_o = L_f \frac{i_{L,peak}}{\Delta t_s T} \Rightarrow i_{L,peak} = \frac{V_o}{L_f} \Delta t_s$$  \hspace{1cm} (51)

$$\therefore I_o = i_{L,peak} \frac{D + \Delta t_s}{2}$$

$$= \frac{V_o T_e}{2L_f} (D + \Delta t_s) \Delta t_s \hspace{1cm} (from \hspace{0.5cm} eq.51)\hspace{1cm}$$

$$= \frac{V_o}{2L_f} D \Delta t_s \hspace{1cm} (from \hspace{0.5cm} eq.47)\hspace{1cm}$$

$$= 4L_{LB,max} D \Delta t_s \hspace{1cm} (from \hspace{0.5cm} eq.45)\hspace{1cm}$$

Hence, $\Delta t_s = \frac{I_o}{4L_{LB,max} D}$  \hspace{1cm} (53)

From equation 47 and equation 53 the ratio $V_o/V_{in}$ is obtained as

$$V_o \over V_{in} = \frac{D^2}{D^2 + \frac{1}{4} \left( I_o / I_{LB,max} \right)}$$  \hspace{1cm} (54)

In Figure 11 the step down characteristics in continuous and discontinuous modes of operation is shown. In this figure the voltage ratio ($V_o/V_{in}$) is plotted as a function of $I_o/I_{LB,max}$ for various duty ratios using equation 32 and equation 54. The boundary between the continuous and the discontinuous mode, shown by dashed line in Figure 11, is obtained using equation 32 and equation 48.
Discontinuous-Conduction Mode with Constant $V_o$

In some applications such as regulated dc power supplies, $V_m$ may vary but $V_o$ is kept constant by adjusting the duty ratio. From equation 44 the average inductor current at the boundary of continuous conduction is obtained as

$$I_{LB} = \frac{TV_o}{2L_f} (1 - D)$$  \hspace{1cm} (56)

From equation 56 it can be seen that, for a given value of $V_o$ the maximum value of $I_{LB}$ occurs at $D = 0$ and is given by

$$I_{LB,\text{max}} = \frac{TV_o}{2L_f}$$  \hspace{1cm} (57)

From equation 56 and equation 57 the relation between $I_{LB}$ and $I_{LB,\text{max}}$ is

$$I_{LB} = (1 - D)I_{LB,\text{max}}$$  \hspace{1cm} (58)

From equation 52, the output current is obtained as

$$I_o = \frac{V_o T}{2L_f} (D + \Delta_1) \Delta_1$$

$$= I_{LB,\text{max}} (D + \Delta_1) \Delta_1 \text{ (from eq.57)}$$

Solving the equation 59 for $\Delta_1$ and substituting its value in equation 47 gives

$$D = \frac{V_o}{V_m} \left( \frac{I_o}{I_{LB,\text{max}}} \right)^{\frac{1}{2}}$$  \hspace{1cm} (60)

References:


Suggested Reading:

Lecture 10: Boost and Buck-Boost Converters

Boost and Buck-Boost Converters

Introduction

The topics covered in this chapter are as follows:

- Principle of Step-Up Operation
- Boost Converter with Resistive Load and EMF Source
- Boost Converter with Filter and Resistive Load
- Buck-Boost Converter

Principle of Step-Up Operation (Boost Converter)

The circuit diagram of a step up operation of DC-DC converter is shown in Figure 1. When the switch $S_1$ is closed for time duration $t_1$, the inductor current rises and the energy is stored in the inductor. If the switch $S_1$ is opened for time duration $t_2$, the energy stored in the inductor is transferred to the load via the diode $D_1$ and the inductor current falls. The waveform of the inductor current is shown in Figure 2.

When the switch $S_1$ is turned on, the voltage across the inductor is

$$v_L = L \frac{di}{dt}$$

The peak to peak ripple current in the inductor is given by

$$\Delta I = \frac{V_L}{L} T_1$$
The average output voltage is
\[ v_0 = V_s + L \frac{\Delta I}{T_2} = V_s \left(1 + \frac{T_1}{T_2}\right) = V_s \frac{1}{1 - D} \]  

(3)

From Equation 3 the following observations can be made:

- The voltage across the load can be stepped up by varying the duty ratio \( D \)
- The minimum output voltage is \( V_s \) and is obtained when \( D = 0 \)
- The converter cannot be switched on continuously such that \( D = 1 \). For values of \( D \) tending to unity, the output becomes very sensitive to changes in \( D \)

For values of \( D \) tending to unity, the output becomes very sensitive to changes in (Fig.3).

**Boost Converter with Resistive Load and EMF Source**

A boost converter with resistive load is shown in Figure 4. The two modes of operation are:

**Mode 1**: This mode is valid for the time duration

\[ 0 \leq t \leq DT \]  

(4)

where \( D \) is the duty ratio and \( T \) is the switching period.

The mode 1 ends at \( t = DT \).
In this mode the switch $S_1$ is closed and the equivalent circuit is shown in Figure 5. The current rises through the inductor $L$ and switch $S_1$. The current in this mode is given by

$$V_s = L \frac{di}{dt}$$  \hspace{1cm} (5)

Since the time instants involved are very small, the term $dt \approx t$. Hence, the solution of Equation 5 is

$$i_1(t) = \frac{V_s}{L} t + I_1$$  \hspace{1cm} (6)

where $I_1$ is the initial value of the current. Assuming the current at the end of mode 1 ($t = DT$) to be $I_2$ ($i_1(t = DT) = I_2$), the Equation 6 can be written as

$$I_2 = \frac{V_s}{L} DT + I_1$$  \hspace{1cm} (7)

![Figure 5: Configuration of a Boost Converter in mode 1](image1)

![Figure 6: Configuration of a Boost Converter in mode 2](image2)

**Mode 2**: This mode is valid for the time duration $DT \leq t \leq T$

In this mode the switch $S_1$ is open and the inductor current flows through the $RL$ load and the equivalent circuit is shown in Figure 6. The voltage equation in this mode is given by

$$V_s = Ri_2 + L \frac{di_2}{dt} + E$$  \hspace{1cm} (9)

For an initial current of $I_2$, the solution of Equation 9 is given by

$$i_2(t) = \frac{V_s - E}{L} \left( 1 - e^{-\frac{R}{L} t} \right) + I_2 e^{-\frac{R}{L} t}$$  \hspace{1cm} (10)

The current at the end of mode 2 is equal to $I_1$:

$$i_2( t = (1-D)t) = I_2 = \frac{V_s - E}{L} \left( 1 - e^{-\frac{R}{L} (1-D)z} \right) + I_2 e^{-\frac{R}{L} (1-D)z}$$  \hspace{1cm} (11)

where $z = TR / L$. 
Solving Equation 7 and Equation 11 gives the values of $I_1$ and $I_2$ as

$$ I_1 = \frac{V_s D z}{R} \frac{e^{-(1-D)z}}{1-e^{-(1-D)z}} + \frac{V_s - E}{R} $$  \hspace{1cm} (12) $$

$$ I_2 = \frac{V_s D z}{R} \frac{1}{1-e^{-(1-D)z}} + \frac{V_s - E}{R} $$  \hspace{1cm} (13) $$

The ripple current is given by

$$ \Delta I = I_2 - I_1 = \frac{V_s}{L} DT $$  \hspace{1cm} (14) $$

The above equations are valid if $E \leq V_s$. In case $E \geq V_s$, the converter works in discontinuous mode.

**Boost Converter with Filter and Resistive Load**

A circuit diagram of a Buck with filter is shown in Figure 7. Assuming that the inductor current rises linearly from $I_1$ to $I_2$ in time $t_1$

$$ V_{in} = L \left( \frac{I_2 - I_1}{t_1} \right) = L \frac{\Delta I}{t_1} \Rightarrow t_1 = \frac{\Delta I}{V_{in}} \frac{L}{t_1} $$  \hspace{1cm} (15) $$

The inductor current falls linearly from $I_2$ to $I_1$ in time $t_2$

$$ V_{in} - V_o = -L \frac{\Delta I}{t_2} \Rightarrow t_2 = L \frac{\Delta I}{V_o - V_{in}} $$  \hspace{1cm} (16) $$

where $\Delta I = I_2 - I_1$ is the peak to peak ripple current of inductor $L$. From equation 15 and equation 16 it can be seen that

$$ \Delta I = \frac{V_{in} t_1}{L} = \frac{(V_o - V_{in}) t_2}{L} $$  \hspace{1cm} (17) $$

![Figure 7: Configuration of a Buck Boost Converter](image-url)
Substituting $t_1 = DT$ and $t_2 = (1 - D)T$ gives the average output voltage

$$V_o = \frac{V_{in}}{t_2} = \frac{V_{in}}{1-D} \Rightarrow (1-D) = \frac{V_{in}}{V_o}$$

(18)

Substituting $D = \frac{t_1}{T} = t_1 f$ into equation 18 gives

$$t_1 = \frac{V_o - V_{in}}{V_o f}$$

(19)

If the boost converter is assumed to be lossless then

$$V_{in} I_{in} = V_o I_o = V_{in} I_o f(1-D)$$

(20)

$$I_{in} = \frac{I_o}{1-D}$$

(21)

The switching period $T$ is given by

$$T = \frac{1}{f} = t_1 + t_2 = \frac{L \Delta I}{V_{in}} + L \Delta I = \frac{\Delta ILV_o}{V_{in} (V_o - V_{in})}$$

(22)

From equation 22 the peak to peak ripple current is given by

$$\Delta I = \frac{V_{in} (V_o - V_{in})}{fLV_o} \Rightarrow \Delta I = \frac{V_{in} D}{fL}$$

(23)

When the switch $S$ is on, the capacitor supplies the load current for $t = t_1$. The average capacitor current during time $t_1$ is $I_c = I_o$, and the peak to peak ripple voltage of the capacitor is

$$\Delta V_c = v_c - v_c (t = 0) = \frac{1}{C} \int_0^{t_1} I_c dt = \frac{1}{C} \int_0^{t_1} I_o dt = \frac{I_o t_1}{C}$$

(24)

Substituting the value of $t_1$ from equation 19 into equation 24 gives

$$\Delta V_c = \frac{I_o (V_o - V_i)}{V_o fC} \Rightarrow \Delta V_c = \frac{I_o D}{fC}$$

(25)

**Condition for Continuous Inductor Current and Capacitor Voltage**

If $I_L$ is the average inductor current, the inductor ripple current is $\Delta I = 2I_L$. Hence, from equation 18 and equation 23 the following expression is obtained

$$\frac{D V_{in}}{fL} = 2I_L = 2I_o = \frac{2V_{in}}{(1-D)R}$$

(26)

The critical value of the inductor is obtained from equation 26 as

$$L = \frac{D(1-D)R}{2f}$$

(27)
If $V_c$ is the average capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2V_a$. Using equation 25 the following expression is obtained

$$\frac{I_oD}{C_f} = 2V_a = 2I_oR$$

(28)

Hence, from equation 28 the critical value of capacitance is obtained as

$$C = \frac{D}{2fR}$$

(29)

**Buck-Boost Converter**

The general configuration of Buck-Boost converter is shown Figure 7. A buck-boost converter can be obtained by cascade connection of the two basic converters:

- the step down converter
- the step up converter

The circuit operation can be divided into two modes:

- During **mode 1** (Figure 8a), the switch $S_i$ is turned on and the diode $D$ is reversed biased. In **mode 1** the input current, which rises, flows through inductor $L$ and switch $S_i$.

- In **mode 2** (Figure 8b), the switch $S_i$ is off and the current, which was flowing through the inductor, would flow through $L$, $C$, $D$ and load. In this mode the energy stored in the inductor ($L$) is transferred to the load and the inductor current ($i_L$) falls until the switch $S_i$ is turned on again in the next cycle.

The waveforms for the steady-state voltage and current are shown in Figure 9.
Buck-Boost Converter Continuous Mode of Operation

Since the switching frequency is considered to be very high, it is assumed that the current through the inductor ($L$) rises linearly. Hence, the relation of the voltage and current in **mode 1** is given by

$$V_{in} = L \frac{I_2 - I_1}{T_1} = L \frac{\Delta I}{T_1}$$

$$\Rightarrow T_1 = L \frac{\Delta I}{V_{in}} \tag{29}$$

The inductor current falls linearly from $I_1$ to $I_2$ in **mode 2** time $T_2$ and is given by

$$V_o = -L \frac{\Delta I}{T_2}$$

$$\Rightarrow T_2 = -L \frac{\Delta I}{V_o} \tag{30}$$

The term $\Delta I = I_2 - I_1$, in **mode 1** and **mode 2**, is the peak to peak ripple current through the inductor $L$. From **equation 29** and **equation 30** the relation between the input and output voltage is obtained as

$$\Delta I = \frac{V_{in} T_1}{L} = -\frac{V_o T_2}{L} \tag{31}$$

The relation between the on and off time, of the switch $S_1$, and the total time duration is given in terms of duty ratio ($D$) as:

$$T_1 = DT \tag{32a}$$

$$T_2 = (1-D)T \tag{32b}$$
Substituting the values of $T_1$ and $T_2$ from equation 32a and equation 32b into equation 31 gives:

$$V_o = \frac{V_{in}D}{1-D}$$

If the converter is assumed to be lossless, then

$$V_{in}I_{in} = -V_oI_o$$

$$V_{in}I_{in} = \frac{V_{in}D}{1-D}I_o \Rightarrow I_{in} = \frac{I_oD}{1-D}$$

The switching period $T$ obtained from equation 29 and equation 30 as:

$$T = T_1 + T_2 = L \frac{\Delta I}{V_o} - L \frac{\Delta I}{V_{in}} = L \Delta I \left( \frac{V_{in} - V_o}{V_{in}V_o} \right)$$

The peak to peak ripple current $\Delta I$ is obtained from equation 35 as

$$\Delta I = \frac{TV_{in}V_o}{L(V_o - V_{in})} = \frac{DT}{L} \frac{V_{in}D}{fL}$$

where

$$f = \text{switching frequency}$$

When the switch $S_1$ is turned on, the filter capacitor supplies the load current for the time duration $T_1$. The average discharge current of the capacitor $I_{cap} = I_{cap}$ and the peak to peak ripple current of the capacitor are:

$$\Delta V_{cap} = \frac{1}{C} \int_{T_1}^{T_2} I_{cap} dt = \frac{1}{C} \int_{T_1}^{T_2} I_o dt = \frac{I_oT_1}{C} = \frac{I_oD}{fC}$$

**Buck-Boost Converter Boundary between Continuous and Discontinuous Conduction**

In Figure 10 the voltage and load current waveforms of at the edge of continuous conduction is shown. In this mode of operation, the inductor current $(i_L)$ goes to zero at the end of the off interval ($T_2$). From Figure 10, it can be seen that the average value of the inductor current is given by

$$I_{LB} = \frac{1}{2} I_2 = \frac{1}{2} \Delta I$$

Substituting the value of $\Delta I$ from equation 36 into equation 38 gives:

$$I_{LB} = \frac{1}{2} \frac{DT}{L} V_{in}$$

In terms of output voltage, equation 39 can be written as

$$I_{LB} = \frac{1}{2} \frac{T}{L} V_o (1-D)$$
The average value of the output current is obtained substituting the value of input current from equation 34 into equation 40 as:

\[ I_{OB} = \frac{1}{2} \frac{T}{L} V_o (1 - D)^2 \]  

(41)

Most applications in which a buck-boost converter may be used require that \( V_{out} \) be kept constant. From equation 40 and equation 41 it can be seen that \( I_{LB} \) and \( I_{OB} \) result in their maximum values at \( D = 0 \) as

\[ I_{LB,max} = \frac{TV_{out}}{2L} \]
\[ I_{OB,max} = \frac{TV_{out}}{2L} \]  

(42)

From equation 38 it can be seen that peak-to-peak ripple current is given by

\[ \Delta I = 2I_{LB} \]  

(43)

Figure 10: Current and voltage waveforms of Buck Boost Converter in boundary between continuous and discontinuous mode

Suggested Reading:

Lecture 11: Multi Quadrant DC-DC Converters I

Multi Quadrant DC-DC Converters I

Introduction
The topics covered in this chapter are as follows:

- Converter classification
- Two Quadrant Converters

Converter Classification
DC-DC converters in an EV may be classified into unidirectional and bidirectional converters. Unidirectional converters are used to supply power to various onboard loads such as sensors, controls, entertainment and safety equipments. Bidirectional DC-DC converters are used where regenerative braking is required. During regenerative braking the power flows back to the voltage bus to recharge the batteries.

The buck, boost and the buck-boost converters discussed so far allow power to flow from the supply to load and hence are unidirectional converters. Depending on the directions of current and voltage flows, dc converters can be classified into five types:

- First quadrant converter
- Second quadrant converter
- First and second quadrant converter
- Third and fourth quadrant converter
- Four quadrant converter

Among the above five converters, the first and second quadrant converters are unidirectional where as the first and second, third and fourth and four quadrant converters are bidirectional converters. In Figure 1 the relation between the load or output voltage \( V_{out} \) and load or output current \( I_{out} \) for the five types of converters is shown.
Second Quadrant Converter

The second quadrant chopper gets its name from the fact that the flow of current is from the load to the source, the voltage remaining positive throughout the range of operation. Such a reversal of power can take place only if the load is active, i.e., the load is capable of providing continuous power output. In Figure 2 the general configuration of the second quadrant converter consisting of a emf source in the load side is shown. The *emf source can be a separately excited dc motor with a back emf of* $E$ and armature resistance and inductance of $R$ and $L$ respectively.

The load current flows out of the load. The load voltage is positive but the load current is negative as shown in Figure 2. This is a single quadrant converter but operates in the second quadrant. In Figure 2 it can be seen that switch $S_4$ is turned on, the voltage $E$ drives current through inductor $L$ and the output voltage is zero. The instantaneous output current and output voltage are shown in Figure 3. The system equation when the switch $S_4$ is on *(mode 1)* is given by

$$0 = L \frac{di_o}{dt} + Ri_o + E$$

(1)
With initial condition \( i_o(t = 0) = I_1 \), gives

\[ i_o = I_1 e^{-\frac{R}{L} t} - \frac{E}{R} \left( 1 - e^{-\frac{R}{L} t} \right) \quad \text{for} \quad 0 \leq t \leq DT \]  

(2)

At time \( t = DT \) the output current is given by reaches a value of \( I_2 \), i.e., \( i_o(t = DT) = I_2 \).

When the switch \( S_4 \) is turned off (mode 2), a magnitude of the energy stored in the inductor \( L \) is returned to the input voltage \( V_{in} \) via the diode \( D_1 \) and the output current \( I_o \) falls. Redefining the time origin \( t = 0 \), the load current is described as

\[ V_{in} = L \frac{di_{out}}{dt} + R i_{out} + E \]  

(3)

At the beginning of mode 2 the initial value of the current is same as the final value of current at the end of mode 1. Hence, the initial condition at the beginning of mode 2 is \( I_2 \).

With this initial condition, the solution of equation 3 is

\[ i_o = I_2 e^{-\frac{R}{L} t} + \frac{V_{in} - E}{R} \left( 1 - e^{-\frac{R}{L} t} \right) \quad \text{for} \quad DT \leq t \leq T \]  

(4)

At the end of mode 2 the load current becomes

\[ i_2(t = T_2 = (1-D)T) = I_3 \]  

(5)

However, at the end of mode 2, the converter enters mode 1 again. Hence, the initial value of current in mode 1 is \( I_3 = I_1 \).

From equation 2 and equation 4 the values of \( I_1 \) and \( I_2 \) is obtained as

\[ I_1 = \frac{V_{in}}{R} \left[ \frac{1 - e^{-(1-D)z}}{1 - e^{-z}} \right] - \frac{E}{R} \]

\[ I_2 = \frac{V_{in}}{R} \left[ \frac{e^{-Dz} - e^{-z}}{1 - e^{-z}} \right] - \frac{E}{R} \]  

(6)

where

\[ z = \frac{TR}{L} \]
Two Quadrant Converters

This converter is a combination of the first and second quadrant converters. Two such converters are discussed here:

- operating in first and second quadrant
- operating in first and fourth quadrant

The following assumptions are made for ease of analysis:

- The input voltage is greater than the load voltage \( V_{in} > E \)
- The positive direction of the current is taken to be the direction from source to load.

First and Second Quadrant Converter

In Figure 4a the configuration of a two quadrant converter providing operation in first and second quadrants is shown.

![Figure 4: First and Second Quadrant Converter](image)

The converter works in **first quadrant** when \( S_2 \) is **off**, diode \( D_2 \) is not conducting and \( S_1 \) is on. If the switch \( S_1 \) is off, \( S_2 \) is on and diode \( D_1 \) is not forward biased, then the converter operates in **second quadrant**. There are four possible modes of operation of this converter. These four possibilities are:

i. **The minimum current \( i_1 > 0 \) and minimum \( i_1 \) and maximum \( i_2 \) currents are positive:** In this mode, only the switch \( S_1 \) and the diode \( D_1 \) operate. When \( S_1 \) is switched **on** at time \( t = 0 \) (Figure 5a), current flows from the source to the motor and the inductor \( L \) gains energy. At time \( t = T \), \( S_1 \) is turned **off** but the current continuous to flow in the same direction and finds a closed path through the load,
the freewheeling diode $D_1$ (Figure 5b). Hence, the instantaneous output current $i_o$ is positive throughout and hence the average output current $I_o$ is also positive. Therefore, the converter operates in **first quadrant**. The waveforms in this condition are shown in Figure 5c.

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**ii. The minimum current $I_1 < 0$, maximum current $I_2 > 0$ and average load current $I_o$ is positive:** In this case the instantaneous load current $i_o$ can be positive or negative but its profile is such that the average load current $I_o$ is positive. In order to analyse the operation of the converter it is assumed that the converter is in steady state. The $S_1$ is turned **on** at $t = 0$, the instantaneous load current is negative ($i_o < 0$) and $D_2$ conducts it (Figure 6a). The drop across the $D_2$, reverse biases $S_1$ thus preventing conduction. The input voltage $V_{in}$ is greater than the load voltage $E$, hence, $\frac{di_o}{dt}$ is positive. When $i_o = 0$, the switch $S_1$ starts conduction and continuous to do so till $T_1$ (Figure 6b). At time $T_1$ the switch $S_1$ is turned off and
the switch $S_2$ is turned on. At this instant the switch $S_2$ cannot conduct because the current is in positive direction. Since the source is isolated, $D_1$ freewheels the inductive current (Figure 6c). The slope $\frac{di_o}{dt}$ being negative, $i_o$ becomes zero after some time and $D_1$ stops conduction. When $i_o$ becomes negative, $S_2$ starts conduction (Figure 6d). This condition remains till time $T$ at which instant $S_1$ is turned on again. The quantities $T_1$ and $T$ are such that the average load current ($I_o$) is positive. The presence of the $S_2$ and $D_2$ facilitate continuous flow of current irrespective of its direction. The current waveforms for this mode of operation are shown in Figure 6e.

![Figure 6a: Load current –ve and $D_2$ conducts](image1)

![Figure 6b: Load current +ve and $S_1$ conducts](image2)

![Figure 6c: Load current +ve and $D_1$ conducts](image3)

![Figure 6d: Load current -ve and $S_2$ conducts](image4)
iii. **The minimum current** $I_1 < 0$, **maximum current** $I_2 > 0$ and **average load current** $I_o$ is negative: The sequence of events for this case is same as case ii except that $T_i$ and $T$ are such that the average load current $I_o$ is negative. Hence, the converter operates in second quadrant. The current waveforms are shown in Figure 7.

iv. $I_2 < 0$: In this case the instantaneous load current is always negative. Hence, the average load current is also negative and the converter operates in the second quadrant. The diode $D_2$ conducts till time $T_i$, $\frac{di}{dt}$ being positive. The current rises from $I_1$ to $I_2$ at $T_i$. The switch $S_2$ starts conduction at $T_i$ and this conduction continuous till $T$, from which moment onwards the sequence repeats. The waveforms are shown in Figure 8.
The following can be observed from the four cases discussed above:

a. For the cases i and iv, during the conduction of $D_2$, $i_o < 0$ but the load voltage $E > 0$ and hence, the load power is negative. This can be interpreted as that the kinetic energy of the motor gets converted into electrical energy and fed back to the source, thereby implying that the motor operates in regenerative braking mode.

b. The switches $S_1$ and $S_2$ can conduct only when their respective triggering signals are present and the instantaneous current through them is positive.
c. The average current through the load is given by

\[ I_o = \frac{V_{in} \left( \frac{T_1}{T} \right) - E}{R} \]

(7)

This current is either positive or negative, respectively, depending on whether

\[ V_{in} \left( \frac{T_1}{T} \right) > E \] \hspace{1cm} or \hspace{1cm} \[ V_{in} \left( \frac{T_1}{T} \right) < E \]

Suggested Reading:


Lecture 12: Multi Quadrant DC-DC Converters II

Multi Quadrant DC-DC Converters II

Introduction

The following topics are covered in this lecture:

- First and Second Quadrant Converter
- Four Quadrant Chopper

First and Fourth Quadrant Converter

In Figure 1a the configuration of two quadrant converter capable of operating in first and fourth quadrants is shown. Both the switches $S_1$ and $S_2$ are turned on for a duration $t = 0$ to $t = T_1$ (Figure 1b) and off for a duration $t = T_1$ to $t = T$. The instantaneous output voltage appearing across the load $v_{out}$ is:

$$v_{out} = V_{in} \quad 0 \leq t \leq T_1$$
$$v_{out} = -V_{in} \quad T_1 \leq t \leq T$$

(1)

When the switches $S_1$ and $S_2$ are turned off, the current through the inductor $L$ continues to flow in the same direction, making the diodes $D_1$ and $D_2$ conduct thus feeding the load energy back to the dc source (Figure 1c). The average load voltage $V_{out}$ is obtained as

$$V_{out} = \frac{1}{T} \left[ \int_0^{T_1} V_{in} \, dt + \int_{T_1}^T (-V_{in}) \, dt \right] = \frac{V_{in}}{T} (T_1 - T_{off})$$

(2)

where

$$T_{off} = T - T_1$$
From **equation 2** it can be seen that for $T_1 > T_{off}$, $V_{out}$ is positive and the current flows from the DC source to load. Both the average load voltage $V_{out}$ and load current $I_o$ being positive, the operation of the converter is in **first quadrant** (Figure 1d). When $T_1 < T_{off}$, $V_{out}$ is negative but $I_o$ is positive and the converter operates in **fourth quadrant** (Figure 1e).
Figure 1d: Waveforms when $T_i > T_{off}$

Figure 1e: Waveforms when $T_i < T_{off}$
Four Quadrant Converters

A four quadrant converter is shown in Figure 2. The circuit is operated as a two quadrant converter to obtain:

a. **Sequence 1**: First and second quadrant operation
b. **Sequence 2**: Third and fourth quadrant operation

**Sequence 1 Operation**

In this mode $S_4$ is kept permanently on. The switches $S_1$ and $S_2$ are controlled as per the following four steps:

- **Mode 1**: If $S_1$ and $S_4$ are turned on, the input voltage $V_{in}$ is applied across the load and current flows in the positive direction from $a$ to $b$. The instantaneous output voltage across the load is $v_{out} = V_{in}$.

- **Mode 2**: When $S_1$ is turned off at time $t = T_1$, the current due to the stored energy of the inductor $L$ drives through $D_2$ and $S_4$ as shown in Figure 3b. The switch $S_2$ is turned on at $t = T_1$ but it does not conduct because it is shorted by $D_2$.

- **Mode 3**: The switch $S_2$ conducts when the current reverses its direction (Figure 3c).
• **Mode 4:** Finally when $S_2$ is turned off at $t = T$, current flows in the negative direction (Figure 3d). The converter operates in the *fourth quadrant* and the power flows from load to source.

![Figure 3a: Mode 1 operation of sequence 1](image1)

![Figure 3b: Mode 2 operation of sequence 1](image2)

![Figure 3c: Mode 3 operation of sequence 1](image3)

![Figure 3d: Mode 4 operation of sequence 1](image4)

The wavforms for *sequence 1* are shown in Figure 4.

![Figure 4: Waveforms for sequence 1](image5)
Sequence 2 Operation

In this sequence, the converter operates in third and fourth quadrant and the switch $S_3$ is permanently kept on. The switches $S_1$ and $S_2$ are controlled as per the following four steps:

- **Mode 1:** $S_2$ is turned on at $t = 0$ but starts conduction only when the current changes sign. The diodes $D_2$ and $D_3$ conduct (Figure 5a) till the current changes its sign. The instantaneous output voltage across the load is $v_{out} = -V_{in}$.

- **Mode 2:** When $S_2$ is turned off at $t = T_1$, the inductor continuous to drive the current in the reverse direction through $S_3$ and $D_1$ (Figure 5b).

- **Mode 3:** The switch $S_1$ is turned on at $t = T_1$ but does not conduct because the current flows in the negative direction and $D_1$ and $S_3$ conduct. Once the current changes the sign $S_1$ and $D_3$ conduct $D_1$ (Figure 5c).

- **Mode 4:** When $S_1$ is turned off at $t = T$, $V_{out} = -V_{in}$ but positive current flows, hence, $D_2$ and $D_3$ conduct $D_1$ (Figure 5d).

The waveforms for this sequence are shown in Figure 6.
Figure 5a: Mode 1 operation of sequence 2

Figure 5b: Mode 2 operation of sequence 2

Figure 5c: Mode 3 operation of sequence 2

Figure 5d: Mode 4 operation of sequence 2

Figure 6: Waveforms for sequence 2
Suggested Reading:


Lecture 13: DC-DC Converters for EV and HEV Applications

DC-DC Converters for EV and HEV Applications

Introduction

The topics covered in this chapter are as follows:

- Multi-input DC-DC Converters
- Multi-input converter Using High/Low Voltage Sources
- Flux Additive DC-DC Converter

Multi-input DC-DC Converters

The rechargeable batteries are the common energy sources for EVs. In order to achieve performance comparable to internal combustion engine vehicle (ICEV), the EVs are powered by an energy source consisting of battery and ultracapacitors. The battery pack supplies the main power and the high power requirements, such as during acceleration, is supplied by supercapacitor bank. Combination of battery and supercapacitor bank enables use of smaller battery pack. In Figure 1 a configuration of an EV with a battery bank and Ultracapacitor bank is shown.
In order to supply the traction motor with two sources, multi-input configuration of D-DC converters are used. The multi input DC-DC converters are classified into following two categories:

- Multi-input Converter Using High/Low Voltage Sources
- Flux additive dc-dc converter.

**Multi-input Converter Using High/Low Voltage Sources**

The Multi input converters can be classified into following types:

- **Type 1: Buck-Buck Converter** (Figure 2a)
- **Type 2: BuckBoost-BuckBoost Converter** (Figure 2b)
- **Type 3: Buck-BuckBoost Converter** (Figure 2c)
- **Type 4: Boost-Boost Converter** (Figure 2d)
- **Type 5: Bidirectional BuckBoost-BuckBoost Converter** (Figure 2e)

![Figure 2a: Type 1: DC DC converter with two input voltages](image1)

![Figure 2b: Type 2: DC DC converter with two input voltages](image2)
Figure 2c: Type 3: DC DC converter with two input voltages

Figure 2d: Type 4: DC DC converter with two input voltages

Figure 2e: Type 5: DC DC converter with two input voltages
In this chapter the functioning of **Type 1** Converter is shown. In the analysis given below, the following assumptions have been made:

- The converter has two input voltage sources $V_1$ and $V_2$.
- The two voltage sources have different magnitude and $V_1 > V_2$.

**Type 1: Buck-Buck Converter**

A DC-DC converter with multi input is shown in **Figure 2a**. By controlling the switching of $S_1$ and switch $S_2$ the power can be extracted from two voltage sources $(V_1, V_2)$ individually or simultaneously.

Based on the switching of $S_1$ and $S_2$, the converter’s operation can be divided into four distinct modes, namely:

**Mode 1:** In this mode of operation, switch $S_1$ is turned on and $S_2$ is turned off. The equivalent circuit for this mode is shown in **Figure 3a**. In this mode the voltage source $V_1$ provides power to the load resistor $R$. The potential across the inductor is

$$e_L = V_1 - V_o \quad (1a)$$

**Mode 2:** In this mode the switch $S_1$ is turned off and the $S_2$ turned on. The equivalent circuit for this mode is shown in **Figure 3b**. In this mode of operation the voltage source $V_2$ charges the inductor $L$ and supplies the load. The potential across the inductor is

$$e_L = V_2 - V_o \quad (1b)$$

**Mode 3:** Both the switches $S_1$ and $S_2$ are turned off. The diodes $D_1$ and $D_2$ provide the current path for the inductor current (**Figure 3c**). The energy stored in $L$ and $C$ is released to the load. The potential across the inductor is

$$e_L = -V_o \quad (1c)$$

**Mode 4:** The switches $S_1$ and $S_2$ are turned on and both the voltage sources $V_1$ and $V_2$ are connected in series and charge the inductor $L$ and supply to the load. The configuration of the circuit in this mode is shown in **Figure 3d**. The voltage across the inductor is

$$e_L = V_1 + V_2 - V_o \quad (1d)$$
The waveforms for the different modes of operation are shown in Figure 4.

![Figure 3a: Mode 1 operation](image)

![Figure 3b: Mode 2 operation](image)

![Figure 3c: Mode 3 operation](image)

![Figure 3d: Mode 4 operation](image)
Having discussed the **Type 1 Multi-input Converter Using High/Low Voltage Sources**, the next section deals with the **Flux additive dc-dc converter**.
Flux Additive DC-DC Converter.

A schematic diagram of a flux additive dc-dc converter is shown in Figure 5 [2]. The converter consists of:

- Two voltage sources
- Three winding coupled transformer
- Common output stage circuit

The converter is fundamentally composed of the buck-boost type dc-dc converter. Based on the switching scheme of the switches, the operation of the converter is divided into 12 modes:

![Figure 5: Flux additive multi input DC-DC converter [2]](image-url)
**Mode 1:** From time \(0 \leq t < t_1\), the switches \(S_2\) and \(S_3\) are turned **off** and the switches \(S_1\) and \(S_4\) are turned **on**. The power flows from the first input stage supplied by voltage source \(V_1\). The input current from the first stage \((i_{in1})\) flows through the transformer \(T_1\), \(D_1\), \(S_1\), \(D_4\) and \(S_4\). The input current of the second stage \((i_{in2})\) freewheels. The magnetic flux produced by \(i_{in1}\) induces emf in the other transformer windings. Due to this induced emf, the current through the output transformer is \(i_{in3}'\). The magnitude of the current \(i_{in2}'\) is **zero** because no closed path is available for the current. Due to the direction of the current \(i_{in3}'\), the diodes \(D_9\) and \(D_{12}\) in the output stage circuit turned on. The equivalent circuit for this mode of operation is shown in **Figure 6a**.

**Mode 2:** In the time interval \(t_1 \leq t < t_2\), the switches \(S_1\), \(S_4\), \(S_5\), \(S_7\) and \(S_8\) are **on**. The equivalent circuit for this mode is shown in **Figure 6b**. The switch \(S_8\) is on but it does not conduct. The input current of the second stage \((i_{in2})\) still freewheels through \(D_5\), \(S_5\), \(D_7\) and \(S_7\). The operations of the first input stage and the output stage circuits remain unchanged.

**Mode 3:** This mode lasts for the time interval \(t_2 \leq t < t_3\). At time \(t = t_2\), the switch \(S_7\) is turned **off**. The equivalent circuit is shown in **Figure 6c**. The current \(i_{in2}\) does not freewheel anymore and flows through \(D_5\), \(S_5\), \(D_8\) and \(S_8\). Operation of first input stage remains unchanged. In this mode, both the input stages transfer power to the output stage. The contribution of both the sources can be explained as follows: since both the currents \((i_{in1}, i_{in2})\) flow through the windings of transformers \(T_1\) and \(T_2\) respectively, the flux linked by the output stage transformer \(T_3\) increases and hence, the current through \(T_3\) is increased resulting in more power flow to the load.
Mode 4: This mode lasts for the time duration $t_3 \leq t < t_4$. At time $t = t_3$ the switches $S_2$ and $S_6$ are turned on. The switches $S_1$ and $S_3$ are still on but do not conduct any current (Figure 6d). The current $i_{in1}$ freewheels through $D_2, S_2, D_4$ and $S_4$, whereas the current $i_{in2}$ freewheels through $D_6, S_6, D_8$ and $S_8$ and no current flows through the transformers $T_1$ and $T_2$. As a result of this, no emf is induced in the transformer $T_3$ and the diodes in the output side ($D_9, D_{10}, D_{11}, D_{12}$) are reverse biased. Hence, no power is transferred from any input stage to the output stage. The power demanded by the load is supplied by the output capacitor $C$.

Mode 5: The duration of this mode is $t_4 \leq t < t_5$. At time $t = t_4$, the switches $S_1$ and $S_3$ are turned off. The current $i_{in1}$ and $i_{in2}$ freewheel and no power is transferred from the sources to the load. The equivalent circuit for this mode is shown in Figure 6e.
Mode 6: This mode lasts for time duration \( t_5 \leq t < t_6 \). At time \( t = t_5 \), the switch \( S_3 \) is turned on. The rest of the circuit behaves as in mode 5 and no power is transferred from the input stage to the output stage. The equivalent circuit is shown in Figure 6f.

![Figure 6c: Mode 3 operation [2]](https://example.com/figure6c)

![Figure 6d: Mode 4 operation [2]](https://example.com/figure6d)

Mode 7: This mode begins at time \( t = t_6 \) and the switch \( S_4 \) is turned on. The equivalent circuit is shown in Figure 6g. The circuit of Figure 6g is similar to that of Figure 6a except that the polarity of the transformer emfs and currents are opposite. Consequently, mode 8 to mode 12 are symmetric to mode 2 to 6. The equivalent circuits of mode 8 to mode 12 are shown in Figure 6h to Figure 6l.
Figure 6e: Mode 5 operation [2]

Figure 6f: Mode 6 operation [2]
Figure 6g: Mode 7 operation [2]

Figure 6h: Mode 8 operation [2]
Figure 6i: Mode 9 operation [2]

Figure 6j: Mode 10 operation [2]
References:


Suggested Reading:


