Modelling of Smart Materials

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LECTURE 9
Modelling of Piezoelectric Material (Part 2)
Organization

- Piezoelectric Coefficients
- A Comparison of Properties
- Actuators Developed using Piezoelectric Material
 PIEZOELECTRIC COEFFICIENTS

- Four constants are frequently used for the comparison of the performances of different piezoelectric materials for sensing and actuation.
- These are Piezoelectric Charge Constant \( d \), Piezoelectric Voltage Constant \( g \), Electro-mechanical coupling factor \( k \) and the frequency constant \( N_p \).
The piezoelectric charge constant \( d \), expressed in ‘m/V’ or ‘pC/N’ (1 Pico-Coulomb (pC) = \( 10^{-12} \) Coulomb), is defined by the following simple relationship:

\[
d_{31} = \frac{\Delta l / l}{V_3 / t} = \frac{q}{F_1}, \quad d_{32} = \frac{\Delta w / w}{V_3 / t} = \frac{q}{F_2}, \quad d_{33} = \frac{\Delta t / t}{V_3 / t} = \frac{q}{F_3}
\]

‘\( q \)’ - charge collected in the electrode surfaces, \( F_i, i=1..3 \), denote the forces along the respective directions & \( V_3 \) denotes the voltage applied along the \( z \) direction.
• The piezoelectric voltage constant, $g$ expressed in V-m/N, is similarly defined as:

$$g_{31} = \frac{V_3}{F_1/w}, \quad g_{32} = \frac{V_3}{F_2/l}, \quad g_{33} = \frac{V_3}{F_3/(w \times l)}$$

$V_3$ is the voltage sensed along the $z$-direction due to the application of pressure. Like $d_{31}$, $g_{31}$ is also usually negative signifying the generation of positive voltage upon the application of tensile force to the system.
• The electro-mechanical coupling factor, \( k \) measures the electro-mechanical energy conversion efficiency. It is expressed by the simple relationship

\[
k^2 = \frac{d_{33}^2}{S^E \varepsilon^T}
\]

Superscripts, \( E \) and \( T \) signify the values of \( S \) and \( \varepsilon \) under constant electric field and constant stress condition, respectively
• When an unconstrained piezoelectric material is subjected to an alternating current, the material shows resonating behaviour at certain frequencies.

• For a disc element of diameter $D_p$, and thickness $t$, the resonating frequencies corresponding to the radial and axial modes ($f_r$ and $f_a$, respectively) are

\[
N_d = f_r D_p, \quad N_t = f_t t
\]
<table>
<thead>
<tr>
<th>Property</th>
<th>PZT (Hard)</th>
<th>PZT(soft)</th>
<th>PZT-PVDF</th>
<th>PMN-PT</th>
<th>LiNbO$_3$</th>
<th>PVDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{33}$ (pC/N)</td>
<td>190</td>
<td>425</td>
<td>120</td>
<td>1240</td>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>$d_{31}$ (pC/N)</td>
<td>-55</td>
<td>-170</td>
<td>-</td>
<td>-</td>
<td>-0.85</td>
<td>-16</td>
</tr>
<tr>
<td>$g_{33}$ mV-m/N</td>
<td>54</td>
<td>27</td>
<td>300</td>
<td>43</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>$g_{31}$ mV-m/N</td>
<td>-16</td>
<td>-11</td>
<td>-</td>
<td>-</td>
<td>-150</td>
<td></td>
</tr>
<tr>
<td>$k_{33}$</td>
<td>0.67</td>
<td>0.70</td>
<td>0.80</td>
<td>0.92</td>
<td>0.17</td>
<td>0.11</td>
</tr>
<tr>
<td>$E_p$ (GPa)</td>
<td>63</td>
<td>45</td>
<td>~30</td>
<td>100</td>
<td>20</td>
<td>2.7</td>
</tr>
<tr>
<td>Density $(\rho)$ (Kg/m$^3$)</td>
<td>7500</td>
<td>7500</td>
<td>3300</td>
<td>8120</td>
<td>4600</td>
<td>1760</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1500</td>
<td>1980</td>
<td>400</td>
<td>3100</td>
<td>1210</td>
<td>700</td>
</tr>
</tbody>
</table>
• Composite of PZT-PVDF has high electro-mechanical coupling with a moderate density which is in between those of PZT and PVDF.

• The elastic modulus is also seen to be quite high in comparison with that of PVDF.

• Thus, such composites present a good trade-off between excellent actuation potential of PZT and sensing capability of PVDF.
• Often, the high-actuation strain generating capability of SMA is exploited by doping elements of SMA into piezo-ceramic PLZST.

• The product is known as shape memory ceramic active material. As high as 6000 μ-strain along with memory effect is achieved through this material.

• However, such materials are still in the developmental stage


Piezo-actuators and sensors

- PZT based actuators can normally generate a maximum strain of about 0.2% (about 2000 $\mu$-strain).
- Single crystals of PZN and PMN may generate strains of the range of 8000 $\mu$-strain, the use of such crystals as actuators are limited due to their high cost and difficulty of integrating in a structure.
Various displacement and force amplification techniques are developed for Piezoelectric actuators:
• The actuators contain multiple piezoelectric elements to get an amplified effect.

• Simplest example - Piezo-stack where many piezoelectric wafers are stacked in such a way that a comparatively larger deformation is obtained in the $d_{33}$ mode by applying a smaller voltage. Other configurations: Rainbow, C-block, and Crescent Forms.
RAINBOW Actuators

- RAINBOWs or Reduced And Internally Biased Oxide Wafers are piezoelectric wafers with an additional heat treatment step to increase their mechanical displacements.

- In the RAINBOW process, typical PZT wafers are lapped, placed on graphite block, and heated in a furnace at 975°C for 1 hour. The heating process causes one side of the wafer to become chemically reduced.

- This reduced layer, approximately 1/3 of the wafer thickness, causes the wafer to have internal strains that shape the once flat wafer into a dome. The internal strains cause the material to have higher displacements and higher mechanical strength than a typical PZT wafer. RAINBOWs with 3 mm of displacements and 10 kg point loads have been reported.
Thunder – Thin Layer Uniform Ferroelectric Driver

**Zero Voltage State**
The piezoceramic is in a compressive prestressed state.
The substrate is under tensile stress.

**Positive Voltage State**
The piezoceramic “shrinks”.
This allows the substrate to flatten and the Thunder attains a flatter structure.

**Negative Voltage State**
The piezoceramic “domes”.
This pulls the substrate and the Thunder arches more.
Externally Leveraged System

• In these actuators mechanical systems are utilized to amplify the output of a piezoelectric actuator – these include actuators like unimorph, bimorph, flexure based actuator, moonie, cymbal etc.
Uni and Bimorph Actuators

**Uni**

**Bimorph**

**Unimorph**

**Bimorph**

- Piezoelectric
- Metal

Deforms when voltage is applied
The principle is based on the deformation of an elliptic shell to amplify the ceramic strain. The ceramic stack is aligned with the great axis of the ellipse. A small deformation of the great axis creates a large displacement of the small axis. The amplification ratio can typically reach 20 times, that means such actuators can reach strokes of 1 mm.
Frequency Leveraged System

- This type of system is based on alternating current supply to a piezo-actuator. Typical examples are piezoelectric inchworm motors, ultrasonic motors, etc.
Piezoceramic inch worm motors

- Linear motors generally used in micro-positioning applications due to the ability to make very small accurate motions.
- There are two clamps and one extensional element. While clamp A (upper clamp) is on and clamp B (lower clamp) is off the drive piezo is extended.
- Then, clamp A is off and B is on returning clamp B to its original position by relaxing the drive piezo. Again, clamp A is on and clamp B is off the drive piezo is extended and so on.
- This is done many times and the rod moves up. Reversing the clamping sequence can make the rod move down.
- These devices can be operated at high frequencies to achieve millimeter per second motions. Some challenges of inch worm devices are achieving high precision in manufacturing so that the clamps work properly.

Actuation for Mobile Micro-Robotics
John C. Tucker, North Carolina State University
Clamp A – Upper Clamp,  Clamp B – Lower Clamp

Inch Worm Process

Step 1
- Clamped
- Not Extended
- Not Clamped

Step 2
- Rod moves up
- Clamped
- Extended
- Not Clamped

Step 3
- Not Clamped
- Extended
- Clamped

Step 4
- Rod moves up
- Not Clamped
- Not Extended
- Clamped
A Cricket Leg Design using Inchworm Motor

- Spring
- Leg
- Pin

1. Motor tightens the spring.
2. Pin releases the leg.
3. Leg moves down rapidly.
References

• Gauzenzi, P., Smart Structures, Wiley, 2009
• Cady, W. G., Piezoelectricity, Dover Publication, 1950
• Crawley, E. F., Intelligent Structures for Aerospace: a technology overview and assessment, AIAA, 33 (8), 1994, pp. 1689-1699
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