Smart Materials, Adaptive Structures, and Intelligent Mechanical Systems

Bishakh Bhattacharya & Nachiketa Tiwari
Indian Institute of Technology Kanpur
Lecture 14
Behavior of Unidirectional Composites
References for this Lecture


Lecture Overview

• Material axes in unidirectional composites

• Constituent volume fraction and its relationship with composite density

• Importance of predictive methodologies used for composite material properties

• Predictive model for longitudinal stiffness

• Predictive model for longitudinal strength
Material axes in unidirectional composites

• A lamina is the building block of modern composite laminated structures.

• Each lamina may have in itself have more than one types of fibers. These fibers may be oriented in different directions.

• A laminate has several layers, or *laminae*.

• Each lamina may have a different:
  – Thickness
  – Fiber orientation angle
  – Fiber material
  – Matrix material

• Understanding the mechanical behavior of a lamina is the first step in understanding mechanics of laminated composite structures.
Material axes in unidirectional composites

• A lamina is also known as a *ply*, or a *layer*.

• Each layer in a laminate has in general three planes of material symmetry. Because of this, they exhibit orthotropic behavior.
  – A lamina cut across these planes of symmetry will exhibit same mechanical properties.

• Lines which are normal to these planes of material symmetry are called *material axes*. These axes and are quite often designated as 1, 2 and 3 axes. These are shown in Fig. 14.1.
  – Axis-1 runs parallel to the direction of fibers, and its direction is called *longitudinal direction*.
  – Axis-2 runs normal to axis-1, but in the plane of lamina. Direction associated with axis-2 is called *transverse direction*. Axis-3 runs normal to axis-1 and axis-2. This is also transverse direction.
  – 1, 2 and 3 are also known as *principal material directions*. 
Material axes in unidirectional composites

• Given the fact that fibers’ strength and stiffness is significantly larger than that of the matrix, a lamina is stiffest and strongest in longitudinal direction.

• Further, in 2 and 3 directions its mechanical properties are roughly the same. In fact, a lamina’s mechanical properties in any direction lying in the 2-3 plane are quite similar.
  – For this reason, a unidirectional lamina is considered as transversely isotropic, i.e. it is isotropic in the 2-3 plane.

• The thickness of a typical carbon or glass fiber ply is 0.127 mm. This thickness depends on number of filaments in a tow. In such plies, fiber diameter may be approximately 10 microns. Each ply may be constructed of yarns or rovings. These are shown in Fig. 14.2.
  – A yarn is a collection of long continuous and interlocked fibers.
  – A roving is a narrow and long bundle of several fibers.
Material axes in unidirectional composites

Fig. 14.1: Principal Material Axes in a Unidirectional Lamina

Fig. 14.2
Roving (left) and Yarn (right)
Failure in Isotropic v/s Transversely Isotropic Materials

• In isotropic materials, failure prediction requires calculating principal stresses or strains and comparing them to allowable stress/strain limits prescribed for the material.

• In non-isotropic materials (e.g. transversely isotropic materials), this approach does not work.
  – The notion of principal stress makes no sense for these materials as material strength changes with direction, and the direction of principal stress may not in most of the cases coincide with direction of maximum strength.

• Thus, for unidirectional materials, we evaluate allowable stress field in context of different strengths of material in principal material directions. These are:
  – Longitudinal tensile strength 
  – Longitudinal compressive strength
  – In-plane shear strength 
  – Lateral tensile strength
  – Lateral compressive strength
Failure in Isotropic v/s Transversely Isotropic Materials

• These five material strength parameters for unidirectional composites are fundamental material properties of a lamina.

• Experimental data shows that these material strength properties of a unidirectional lamina are mutually independent, particularly at macro-scale.

• Hence, if we are able to calculate stress-field in a unidirectional lamina using 1-2-3 axes as reference frame, then we can predict failure in such lamina.
Volume and Mass Fraction

• The relative proportions of fiber and matrix have a significant influence on the mechanical properties of composite lamina.

• These proportions can be expressed either as volume fractions, or as mass fractions. While mass fractions are easier to obtain during fabrication of composites, volume fractions are handier in theoretical analyses of composites.

• If $v_c$, $v_m$, and $v_f$, are volumes of composite, matrix, and fiber, respectively, then volume fraction of matrix ($V_m$) and fiber ($V_f$) are defined below.
  
  \[ V_m = \frac{v_m}{v_c} \quad \text{and} \quad V_f = \frac{v_f}{v_c} \quad \text{where,} \quad v_c = v_m + v_f \]

• And if $m_c$, $m_m$, and $m_f$, are masses of composite, matrix, and fiber, respectively, then mass fraction of matrix ($M_m$) and fiber ($M_f$) are defined below.

  \[ M_m = \frac{M_m}{M_c} \quad \text{and} \quad M_f = \frac{M_f}{M_c} \quad \text{where,} \quad m_c = m_m + m_f \]
Volume and Mass Fraction

• Using volume fractions we can now calculate the overall density of the composite. If \( \rho_m \), \( \rho_f \), and \( \rho_c \) are densities of matrix, fiber and composite, respectively, then density of composite (\( \rho_c \)) can be calculated as shown below.

\[
\begin{align*}
  m_c &= m_m + m_f \\
  \rho_c \nu_c &= \rho_m \nu_m + \rho_f \nu_f
\end{align*}
\]

• Dividing this relation by volume of composite, we can write:

\[
\begin{align*}
  \rho_c &= \rho_m \nu_m / \nu_c + \rho_f \nu_f / \nu_c \\
  \rho_c &= \rho_m V_m + \rho_f V_f
\end{align*}
\]  
(Eq. 14.1)

• Similarly, we can also develop a relation for composite’s density in terms of weight fractions and densities of fiber and matrix.
What you learnt in this lecture?

• Material axes of a lamina, and the notion of transverse isotropy

• How to calculate density of a composite, using volume and mass fraction values.

• The rationale underlying development of predictive methodologies for estimating composite material properties

• Relations to predict longitudinal modulus of a unidirectional lamina

• Relations to predict strength of unidirectional lamina

• Different failure criterion for a unidirectional lamina which undergoes pure uniform tension