

Module 7 Design of Springs

Lesson 3

Design of Leaf Springs

Instructional Objectives:

At the end of this lesson, the students should be able to understand:

- Working of leaf spring
- Types of leaf springs
- Design theme of leaf springs
- Laminated spring and its modifications

7.3.1 Leaf Springs

In order to have an idea of working principle of a leaf spring, let us think of the diving board in a swimming pool. The diving board is a cantilever with a load, the diver, at its free end. The diver initiates a to and fro swing of the board at the free end and utilizes the spring action of the board for jumping. The diving board basically is a leaf spring.

The leaf springs are widely used in suspension system of railway carriages and automobiles. But the form in which it is normally seen is laminated leaf spring.

A simple cantilever type leaf spring is shown in the Fig. 7.3.1.

In the cantilever beam type leaf spring, for the same leaf thickness, h , leaf of uniform width, b (case 1) and, leaf of width, which is uniformly reducing from b (case 2) is considered. From the basic equations of bending stress and deflection, the maximum stress, σ_{\max} and tip deflection, δ_{\max} can be derived.

For case 1(uniform width)

$$\begin{aligned}\sigma_{\max} &= \frac{6FL}{bh^2} \\ \delta_{\max} &= \frac{4FL^3}{Eb^3}\end{aligned}\quad (7.3.1)$$

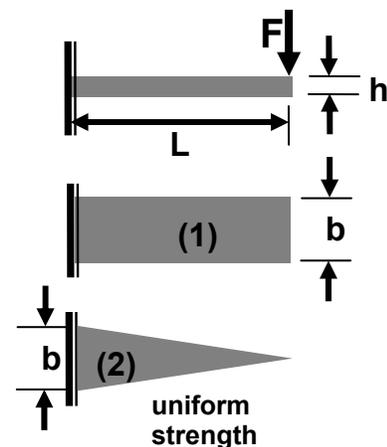


Fig.7.3.1

Where, E is the Elastic modulus of the spring material.

For case 2(non uniform width)

$$\sigma_{\max} = \frac{6FL}{bh^2}$$

$$\delta_{\max} = \frac{6FL^3}{Ebh^3} \quad (7.3.2)$$

In the second case it is observed that instead of uniform width leaf, if a leaf of varying width (triangular one as shown in the figure) is used, the bending stress at any cross section is same and equal to σ_{\max} . This is called as leaf of a uniform strength. Moreover, the tip deflection being more, comparatively, it has greater resilience than its uniform width counterpart. Resilience, as we know, is the capacity to absorb potential energy during deformation. However, one should keep

in mind that in order to withstand the shear force the tip has to have some width. This is shown as a red zone in the figure. In one way non uniform width leaf is a better design than a uniform width leaf.

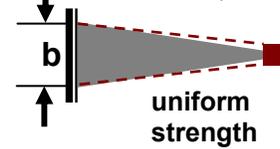


Fig. 7.3.2

Leaf spring of simply supported beam type is shown in the Fig. 7.3.3, for which the stress and deflection equation are also given as in the case of cantilever.

For case 1(uniform width)

$$\sigma_{\max} = \frac{3FL}{bh^2}$$

$$\delta_{\max} = \frac{2FL^3}{Ebh^3} \quad (7.3.3)$$

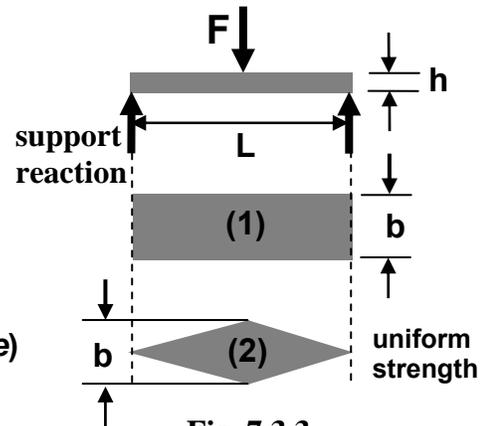


Fig. 7.3.3

For case 2(non uniform width *Lozenge-shape*)

$$\sigma_{\max} = \frac{3FL}{bh^2}$$

$$\delta_{\max} = \frac{3FL^3}{Ebh^3} \quad (7.3.4)$$

One of the applications of leaf spring of simply supported beam type is seen in automobiles, where, the central location of the spring is fixed to the wheel axle. Therefore, the wheel exerts the force F (opposite to the direction shown in the figure), on the spring and support reactions at the two ends of the spring come

from the carriage. The diamond shaped leaf, shown as case 2, is named as Lozenge shape and it is again a beam of uniform strength.

7.3.2 Design theme of a leaf spring

Let us consider the simply supported leaf of Lozenge shape for which the maximum stress and maximum deflection are known. From the stress and deflection equations the thickness of the spring plate, h , can be obtained as,

$$\mathbf{h} = \frac{\sigma_{\max} \mathbf{L}^2}{\mathbf{E} \delta_{\max}} = \frac{\sigma_{\text{des}} \mathbf{L}^2}{\mathbf{E} \delta_{\text{des}}} \quad (7.3.5)$$

The σ_{\max} is replaced by design stress σ_{des} . Similarly, δ_{\max} is replaced by δ_{des} . \mathbf{E} is the material property and depends on the type of spring material chosen. \mathbf{L} is the characteristic length of the spring. Therefore, once the design parameters, given on the left side of the above equation, are fixed the value of plate thickness, \mathbf{h} can be calculated.

Substitution of \mathbf{h} in the stress equation above will yield the value of plate width \mathbf{b} .

$$\mathbf{b} = \frac{3\mathbf{FL}}{\sigma_{\text{des}} \mathbf{h}^2} \quad (7.3.6)$$

In the similar manner \mathbf{h} and \mathbf{b} can be calculated for leaf springs of different support conditions and beam types.

7.3.3 Laminated Springs

One of the difficulties of the uniform strength beam, say Lozenge shape, is that the value of width \mathbf{b} sometimes is too large to accommodate in a machine assembly. One practice is that instead of keeping this large width one can make several slices and put the pieces together as a laminate. This is the concept of laminated spring. The Fig.7.3.4 shows the concept of formation of a laminated spring.

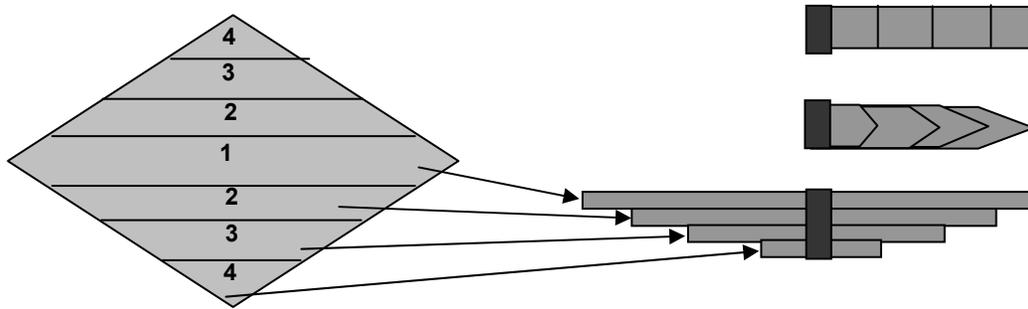


Fig. 7.3.4

Laminated Spring

The Lozenge shaped plate is cut into several longitudinal strips, as indicated in the figure. The central strip, marked 1 is the master leaf which is placed at the top. Then two pieces, marked 2 are put together, side by side to form another leaf and placed below the top leaf. In the similar manner other pairs of strips, marked 3 and 4 respectively are placed in the decreasing order of strip length to form a laminated spring. Here width of each strip, b_N is given as,

$$(7.3.7) \quad b_N = \frac{b}{N}, \quad \text{Where } N \text{ is the number of strips}$$

In practice, strips of width, b_N and lengths, say equal to strip1, strip2 etc., as shown in the example, are cut and put in the laminated form. The stress and deflection equations for a laminated spring is,

$$\sigma_{\max} = \frac{pFL}{Nb_N h^2} \quad \text{and} \quad \delta_{\max} = \frac{qFL^3}{ENb_N h^3} \quad (7.3.8)$$

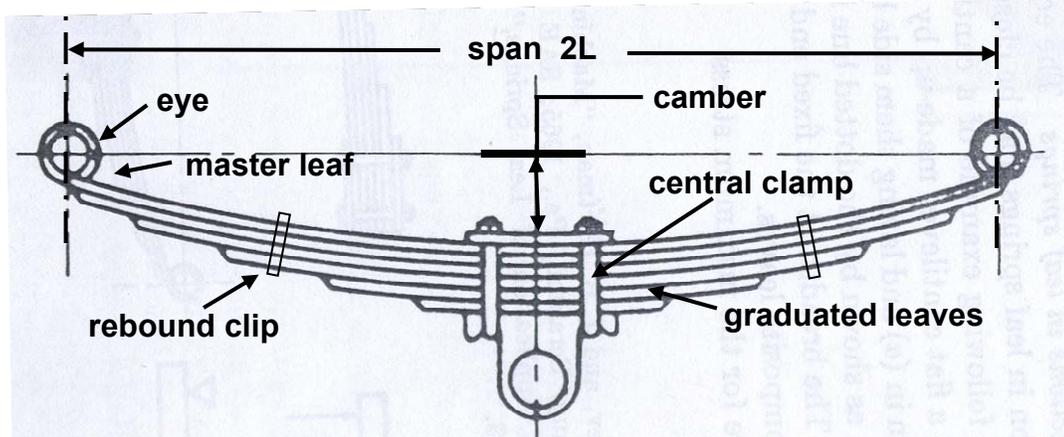
Where, constants p and q are given as,

	p	q
Simply supported beam :	3	3
Cantilever beam :	6	6

It is to be noted that the ends of the leaves are not sharp and pointed, as shown in figure. In fact they are made blunt or even made straight to increase the load bearing capacity. This change from ideal situation does not have much effect on the stress equation. However, small effect is there on the deflection equation.

In the following section we will discuss about few more constructional details of a laminated leaf spring.

7.3.4 Laminated semi-elliptic spring



Laminated semi-elliptic spring

Fig 7.3.5

The Fig 7.3.5 shows a laminated semi-elliptic spring. The top leaf is known as the master leaf. The eye is provided for attaching the spring with another machine member. The amount of bend that is given to the spring from the central line, passing through the eyes, is known as camber. The camber is provided so that even at the maximum load the deflected spring should not touch the machine member to which it is attached. The camber shown in the figure is known as positive camber. The central clamp is required to hold the leaves of the spring. However, the bolt holes required to engage the bolts to clamp the leaves weaken the spring to some extent. Rebound clips help to share the load from the master leaf to the graduated leaf.

7.3.5 Materials for leaf spring

Materials for leaf spring are not as good as that for the helical spring.

Plain carbon steel, Chromium vanadium steel, Chromium- Nickel- Molybdenum steel, Silicon- manganese steel, are the typical materials that are used in the design of leaf springs.

7.3.6 Standard sizes of leaf spring

Width (mm) : 25-80 mm in steps of 5mm
Thickness (mm) : 2-8 mm in steps of 1mm, 10-16 mm in steps of 2mm

In order to carry heavy load few more additional full length leaves are placed below the master leaf for heavy loads. Such alteration from the standard laminated leaf spring, what we have learnt above, does not change the stress value, but deflection equation requires some correction.

$$\delta_{\max} = \frac{\delta_c qFL^3}{ENb_N h^3} \quad (7.3.9)$$

Where, correction in deflection, δ_c is given as,

$$\delta_c = \frac{1.0 - 4m + 2m^2 \{1.5 - \ln(m)\}}{(1.0 - m)^3}$$

where,

$$m = \frac{N_f}{N}$$

N_f = Number of full length leaves

N = Total number of leaves in the spring

7.3.7 Stresses due to support hinges

The master leaf of a laminated spring is hinged to the supports. The support forces induce, stresses due to longitudinal forces and stresses arising due to possible twist. Hence, the master leaf is more stressed compared to other the graduated leaves. Methods to reduce additional stresses could be,

1. Master leaf is made of stronger material than the other leaves.
2. Master leaf is made thinner than the other leaves. This will reduce the bending stress as evident from stress equation.
3. Another common practice is to increase the radius of curvature of the master leaf than the next leaf.

The last method is explained through Fig 7.3.6.

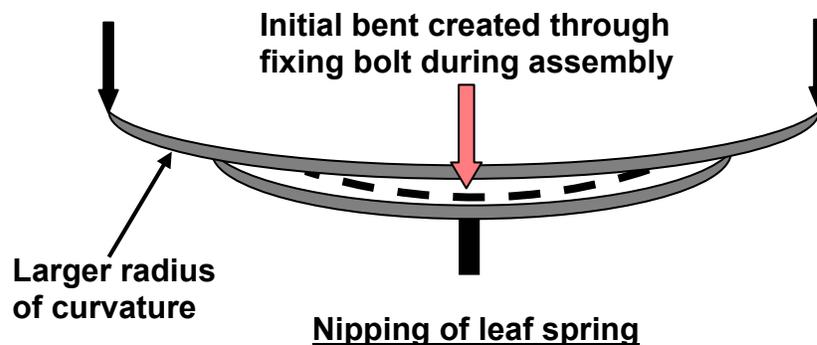


Fig 7.3.6 Version 2 ME , IIT Kharagpur

The master leaf has a larger radius of curvature compared to the additional leaf that is placed below so obviously a gap will be created between the two leaves as indicated in the figure. Now, an initial bent is created during assembly by tightening the central bolt. Therefore, some amount of compressive stress will be produced at the inside curvature of the master leaf. Similarly, at the outside curvature of the master leaf tensile stress will be produced. Both these stresses are initial stresses in the master leaf. However, by such operation of tightening the central bolt, the additional leaf that is placed beneath the master leaf has a tendency to flatten out and as a result the stress pattern of the additional leaf will be reverse of that of the master leaf, tensile stress is produced at the inner curvature and compressive stress is produced at the outer curvature. Hence, when the spring is loaded, for both the master leaf and the additional leaf, tensile stress will be produced at the inner curvature and compressive stress will be produced at the outer curvature. Therefore, due to opposite nature of initial stress and loading stress, the master leaf will experience lesser stress on both the surfaces. However, due to same nature of initial stress and loading stress, the additional leaf is stressed more compared to the master leaf. But, it is to be noted that the higher stress on the additional leaf is actually shared between all other leaves than the master leaf. This practice of stress relief in the master leaf is known as *Nipping of leaf spring*. As a matter of fact, all the leaves of a laminated leaf spring do have certain amount of nipping, so that there will be gaps between the leaves, as a result the stresses will be uniformly distributed and accumulated dusts can also be cleaned.

Sample problem

Design a leaf spring to carry a load of 3400N and placed over a span of 800 mm. The spring can deflect by 50mm. Consider, allowable bending stress for the spring material as 350 MPa and $E=2 \times 10^5$ MPa.

Let us consider the design to be based on uniform strength beam. Then from section 7.3.2 we find that,

$$\text{Leaf thickness, } h = \frac{\sigma_{des} L^2}{E \delta_{des}} = \frac{350 \times 400^2}{2 \times 10^5 \times 50} = 5.6 \text{ mm} \approx 6 \text{ mm}$$

$$\text{Leaf width, } b = \frac{3FL}{\sigma_{des} h^2} = \frac{3 \times 3400 \times 400}{350 \times 50^2} \approx 324 \text{ mm}$$

It is observed that the width is too large to accommodate as a machine member. Hence, if we consider, say 6 springs, then width of each spring becomes 54mm.

Questions and answers

Q1. What are the forms of leaf spring ?

A1. Leaf springs are of two forms: cantilever and simply supported type.

Q2. What does the term “uniform strength” in the context of leaf spring mean?

A2. If the leaf spring has a shape of uniformly varying width (say Lozenge shape) then the bending stress at all section remains uniform. The situation is also identical as before in case of varying thickness, the thickness should vary non-uniformly with length to make a beam of uniform strength ($L/h^2 = \text{constant}$). These leaves require lesser material, have more resilience compared to a constant width leaf. These types of springs are called leaf springs of uniform strength.

Q3. What is “nipping” in a laminated spring?

A3. In general the differential curvature between the master leaf and the next leaves is provided in a laminated spring, where, radius of curvature being more for the master leaf. This construction reduces the stress in the master leaf as compared to the other leaves of the spring in a laminated spring. This type of constructional feature is termed as *nipping*.

References

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