Module 2 - GEARS

Lecture 17 – DESIGN OF GEARBOX

Contents

17.1 Commercial gearboxes
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17.1 COMMERCIAL GEARBOXES

Various commercial gearbox designs are depicted in Fig. 17.1 to 17.10. These include single to multistage ranging from spur, helical, bevel to worm gears.

Fig. 17.1 Commercial Gearbox Design

Fig. 17.2 Two stage helical gearbox
Fig.17.3 A single stage bevel gearbox

Fig.17.4 Worm gearbox
Fig. 17.5 Worm gearbox, sectional front and side views

Fig. 17.6 Worm gearbox, without cooling fins, sectional front and side views
Fig.17.7 Helical gear lubrication with idler gear

Fig.17.8 Spur gear lubrication with stream by nozzles

Fig.17.9 A double reduction spur gear box
17.2 HELICAL GEARBOX DESIGN - PROBLEM 1

In a turbine drive 300 kW power is transmitted using a pair of double helical gear. The pinion speed is 2950 rpm and that of the gear is about 816.5 rpm. There are no space constraints on the gear drive. Selecting suitable materials, design the pinion and the gear to last for $10^8$ cycles. Design the gearbox completely.

**Data:** $W = 300kW; n_1 = 2950rpm; n_2 \approx 816.5$ rpm; Desired Life $10^8$ cycles.

**Solution:**

1. Angular speed of the input shaft

$$\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi \times 2950}{60} = 308.77 \text{ rad/s}$$

2. Torque:

$$T_1 = \frac{1000W}{\omega} = \frac{1000 \times 300}{308.77} = 971.6 \text{Nm}$$

3. The details of the gear design carried out are given in Table 1 and 2.
The final specifications of the pinion and gear are:
20° pressure angle involute teeth with helix angle of 35°, \( h_a = 1m_n \), \( h_r = 1.25m_n \).
\[ i = \frac{Z_2}{Z_1} = \frac{105}{29} = 3.62 \]

**Table 17.1 Gear dimensions**

<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>( m_n ) mm</th>
<th>d mm</th>
<th>( d_a ) mm</th>
<th>( d_b ) mm</th>
<th>( d_r ) mm</th>
<th>( m_t ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion</td>
<td>29</td>
<td>5</td>
<td>177.01</td>
<td>187.01</td>
<td>161.76</td>
<td>164.51</td>
<td>6.104</td>
</tr>
<tr>
<td>Gear</td>
<td>105</td>
<td>5</td>
<td>640.92</td>
<td>650.92</td>
<td>585.69</td>
<td>628.42</td>
<td>6.104</td>
</tr>
</tbody>
</table>

**Table 17.2a Gear specifications**

<table>
<thead>
<tr>
<th>Element</th>
<th>( \Phi_n )</th>
<th>( \varphi )</th>
<th>B mm</th>
<th>( p_t ) mm</th>
<th>( p_a ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion</td>
<td>20°</td>
<td>23.96°</td>
<td>70</td>
<td>19.165</td>
<td>27.37</td>
</tr>
<tr>
<td>Gear</td>
<td>20°</td>
<td>23.96°</td>
<td>70</td>
<td>19.165</td>
<td>27.37</td>
</tr>
</tbody>
</table>

**Table 17.2b Gear specifications**

<table>
<thead>
<tr>
<th>Element</th>
<th>CR(_t)</th>
<th>CR(_a)</th>
<th>CR</th>
<th>FS ( s_b )</th>
<th>FS ( s_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion</td>
<td>1.3044</td>
<td>1.2787</td>
<td>2.583</td>
<td>1.99</td>
<td>1.73</td>
</tr>
<tr>
<td>Gear</td>
<td>1.3044</td>
<td>1.2787</td>
<td>2.583</td>
<td>1.89</td>
<td>1.53</td>
</tr>
</tbody>
</table>
Fig. 17.11 (a) Pinion

Fig. 17.11 (b) Gear

All dimensions are in mm and not to scale

Fig. 17.12 A commercial double helical gearbox
All dimensions are in mm

4. Shaft design is based on the ASME equation:

Tangential load on the shaft: \( F_t = \frac{T}{r} = \frac{971.6}{0.088.5} = 11kN \)

\( F_r = F_t \tan \theta = 11 \tan 23.96^\circ = 4.89kN \)

\( F = (F_t^2 + F_r^2)^{0.5} \)
\( = (11^2 + 4.89^2)^{0.5} = 15.42kN \)

Bending moment at C

\( M = \frac{FL}{4} = 15.42 \times 0.15/4 = 0.58 \text{ kNm} \)

By ASME code equation for shaft design we have,

\[
d = \frac{16}{\pi (1-k) \tau} \sqrt{(K_b M)^2 + (K_i T)^2}
\]
k = 0.2 i.e, 20% reduction in strength due to keyway is assumed. From Table 17.3, for rotating shaft with minor shock loads, \( K_b = 1.5 \) and \( K_t = 1.0 \).

Taking C45 steel for the shaft, \( \sigma_{yp} = 360 \text{ MPa} \)

\( \tau_{yp} = \frac{\sigma_{yp}}{2} = \frac{360}{2} = 180 \text{ MPa} \) and taking factor of safety of \( [\tau] = \frac{\tau_{yp}}{2} = \frac{180}{2} = 90 \text{ MPa} \)

**Table 17.3 Combined shock and fatigue factors for ASME code shaft design equation**

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>( K_b )</th>
<th>( K_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary shaft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gradually applied load</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Suddenly applied load</td>
<td>1.5-2.0</td>
<td>1.5-2.0</td>
</tr>
<tr>
<td>Rotating shaft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gradually applied load</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Suddenly applied load</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With minor shocks</td>
<td>1.5-2.0</td>
<td>1.0-1.5</td>
</tr>
<tr>
<td>With heavy shocks</td>
<td>2.0-3.0</td>
<td>1.5-3.0</td>
</tr>
</tbody>
</table>

\[
d = \frac{16}{\pi(1-k)[\tau]} \sqrt{(K_mM)^2 + (K_tT)^2}
\]

\[
d = \frac{16 \times 10^6}{\pi(1-0.2) \times 90} \sqrt{(1.5 \times 0.58)^2 + (1 \times 0.9716)^2} = 46 \text{ mm}
\]

Take \( d = 50 \text{ mm} \). Check for deflection at the pinion centre.

Deflection at C:

\[
\delta = \frac{F \theta}{48EI} = \frac{15420 \times 150}{48 \times 2.1 \times 10^6} \left( \frac{\pi \times 50^4}{64} \right) = 0.017 \text{ mm}
\]

Since \( \delta < 0.01 \text{ m} = 0.01 \times 5 = 0.05 \text{ mm} \), the design is OK.

Check for slope at the bearing at A.
Slope: \( \alpha = \frac{FL^2}{16EI} = \frac{15420 \times 150^2}{16 \times 2.1 \times 10^5 \times x (\frac{\pi \times 50^4}{64})} = 0.00034 \text{ rad.} \)

\( \alpha < 0.0008 \text{ rad.} \) Hence the design is OK.

5. Check for the pinion size. The minimum pitch diameter of the pinion should be

- \( d_{1\min} \geq 2 \times \text{bore} + 0.25 \text{ m} \)
- where \( d \) is the bore diameter and \( m \) is the module expressed in mm.

\( D_{1\min} \geq 2 \times \text{bore} + 0.1 \times m = 2 \times 50 + 0.1 \times 5 = 100.5 \text{ mm} \)

Since \( d_1 = 177.01 \text{ mm} > D_{1\min} \). The design is satisfactory. Pinion drawing is shown in Fig. 17.16 with full dimensions.

6. The outside diameter of the hubs in larger gears should be 1.8 times the bore for steel. The hub length should be at least 1.25 times the bore and never less than the width of the gear.

Gear shaft diameter = \( d (i)^{1/3} = 50 (3.62)^{1/3} = 77 \text{ mm.} \)

Gear shaft diameter of 80 mm is taken.

The hub diameter: \( d_H = 1.8 \times 80 = 144 \text{ mm}, 150 \text{ mm is taken.} \)

Hub length is taken as \( L = 1.25d = 1.25 \times 80 \approx 100 \text{ mm} \)

Other dimensions of the gear are given in Fig. 17.17.

In view of the dimensions of the pinion and the gear, the dimensions of the shaft layout is revised as shown in Fig. 17.18 When the calculations are redone, there is no change in shaft diameters. The same diameters are adopted for further computations.
Fig. 17.16. Pinion blank drawing

Fig. 17.17. Gear blank drawing

Table 17.4 Dimensions of pinion and gear blank

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>50 mm</td>
<td>80 mm</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$1.8d \approx 90\text{ mm}$</td>
<td>$1.8d \approx 150\text{ mm}$</td>
</tr>
<tr>
<td>B</td>
<td>70 mm</td>
<td>70 mm</td>
</tr>
<tr>
<td>C</td>
<td>$0.3B \approx 22\text{ mm}$</td>
<td>$0.3B \approx 22\text{ mm}$</td>
</tr>
<tr>
<td>$D_e$</td>
<td>187.01 mm</td>
<td>650.92 mm</td>
</tr>
<tr>
<td>m</td>
<td>5 mm</td>
<td>5 mm</td>
</tr>
<tr>
<td>$D_o$</td>
<td>$\approx D_e - 10m = 137\text{ mm}$</td>
<td>$\approx D_e - 10m = 600\text{ mm}$</td>
</tr>
<tr>
<td>n</td>
<td>0.5 m = 2.5 mm</td>
<td>0.5 m = 2.5 mm</td>
</tr>
<tr>
<td>$D_1$</td>
<td>$(D_o + d_1) / 2 = 114\text{ mm}$</td>
<td>$(D_o + d_1) / 2 = 375\text{ mm}$</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$\approx(D_o - d_1) / 5 = 10\text{ mm}$</td>
<td>$\approx(D_o - d_1) / 5 = 90\text{ mm}$</td>
</tr>
<tr>
<td>L</td>
<td>1.25 $d = 70\text{ mm}$</td>
<td>$1.25d \approx 100\text{ mm}$</td>
</tr>
<tr>
<td>W</td>
<td>0.25 $d = 12.5\text{ mm}$</td>
<td>0.25$d = 20\text{ mm}$</td>
</tr>
<tr>
<td>Keyway depth</td>
<td>0.125$d \approx 6\text{ mm}$</td>
<td>0.125$d \approx 10\text{ mm}$</td>
</tr>
</tbody>
</table>
7. Bearings selection is based on 90% reliability for the following life:
8 hrs. Operation per day life = 20,000-30,000 hrs.
Consider the bearings at A & B with Life = 30,000 hrs,
P = 15420 / 2 = 7710 N,
\(f_n = 0.224\) for \(n = 2950\) rpm from FAG catalog.
\(f_L = 3.91\) for 30000 hrs life assuming 16 hrs/day working from FAG catalog.
\(C = \frac{f_L}{f_n} P = \frac{3.91}{0.224} \times 7710 = 134581\) N = 134.6 kN

Giving 2.5 mm abutment for the bearings, shaft diameter of the bearing should be 45 mm.
Roller bearing NJ 2309 satisfies this requirement
\(C = 137\) kN, \(C_o = 153\) kN, \(d_o = 100\) mm, \(d_i = 45\) mm, \(b = 36\) mm.

For the gear shaft of diameter 80mm, giving abutment of 2.5 mm, bearing bore diameter should be 75 mm.
P = 7710 N,
\(f_n = 0.345\) for shaft speed of 815 rpm.
f_L = 3.91 for Life of 30,000 hrs.

C = (f_L / f_n) x P = (3.91 / 0.345) x 7710 = 87,380 = 87.38 kN

Deep groove ball bearing 6315 with C=114kN, C_o=67kN; d_o = 160 mm; d_i = 75mm; b=37mm.

8. Gearbox dimensions are fixed based on thumb rule given in Table 17.5.

Table 17.5 Wall thickness ‘s’ in mm of the gearboxes

<table>
<thead>
<tr>
<th>Material</th>
<th>Non-case hardened gears</th>
<th>Case hardened gears</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI castings</td>
<td>0.007L + 6 mm</td>
<td>0.010 L + 6 mm</td>
</tr>
<tr>
<td>Steel castings</td>
<td>0.005L + 4 mm</td>
<td>0.007L + 4 mm</td>
</tr>
<tr>
<td>Welded construction</td>
<td>0.004L + 4 mm</td>
<td>0.005L + 4 mm</td>
</tr>
</tbody>
</table>

Where L is the largest dimension of the housing in mm.

s = 0.005L + 4 mm = 0.005 x 900 + 4 ≈ 10 mm

Top cover thickness: s_c = 0.8s = 8 mm.

Flange thickness: s_f = 2s = 2x10 = 20 mm

Flange cover bolt diameter: d_c_b = 1.5s ≈ 16 mm M16 bolts.

Bolt spacing: 6d = 6x16 ≈ 100 mm

Foundation bolt diameter: d_f_b = (2T)^(1/3) ≥ 12 mm
d_f_b = (2x3.62x971.6)^(1/3) = 19.2, Take M20 bolts.

The thickness of the foundation flange should be:
S_f_f ≥ 1.5 d_f_b = 1.5x20 = 30 mm
The width of the flange at the base: \( w_b = 2.5d = 2.5 \times 20 = 50 \text{ mm} \)

The width of the flanges at the two halves of the housing should be:
\( w_f = 2.5d = 2.5 \times 16 = 40 \)

With welding bead of 5mm, \( w_f = 45 \text{ mm} \) is taken.

Outside dimension of the bearing housing 1.2-1.5 times outside diameter of the bearing.
Bearing housing diameters are: 1.5x100 = 150mm and: 1.3x160 = 210 mm taking 6 Nos. M10 bolts for the bearing covers.
The views of the bottom and top half of the gearbox are shown in Fig. 17.19 and Fig.17.20.
RECOMMENDED OIL FOR VARIOUS SLIDING SPEEDS

Table 17.6 Recommended oil viscosity V50 [cSt at 50°C] for different sliding speeds

<table>
<thead>
<tr>
<th>v(m/s)</th>
<th>0.25</th>
<th>0.4</th>
<th>0.63</th>
<th>1.0</th>
<th>1.6</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>V50 min</td>
<td>175</td>
<td>145</td>
<td>120</td>
<td>100</td>
<td>83</td>
<td>69</td>
</tr>
<tr>
<td>V50 max</td>
<td>350</td>
<td>290</td>
<td>240</td>
<td>200</td>
<td>166</td>
<td>138</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>v(m/s)</th>
<th>4.0</th>
<th>6.3</th>
<th>10</th>
<th>16</th>
<th>25</th>
<th>40</th>
<th>63</th>
</tr>
</thead>
<tbody>
<tr>
<td>V50 min</td>
<td>57</td>
<td>47</td>
<td>39</td>
<td>32</td>
<td>27</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>V50 max</td>
<td>114</td>
<td>94</td>
<td>78</td>
<td>64</td>
<td>54</td>
<td>44</td>
<td>36</td>
</tr>
</tbody>
</table>
The gears are operating at a sliding speed of \( v = \omega r = 308.77 \times 0.0885 = 27.33 \text{ m/s} \).

From the Table 17.6, the recommended oil viscosity at 50°C for this operation is \( V_{50} \) between 25 to 51 cSt (interpolated values). ISO VG 100 satisfies this, see Fig. 17.21.

The equivalent grade from chart in Fig. 17.22, SAE 30 oil comes under this range and is recommended for the operation.

**ISO VG GRADE LUBRICANTS**

![Fig.17.21 Viscosity – Temperature curve for ISO VG graded oils](image-url)
SAE OIL VISCOSITY CHART

Fig. 17.22 Viscosity – Temperature curves of SAE graded oils

9. Losses in gear boxes:

Total power loss \( L = L_t + L_{ch} + L_b + L_s \)

- \( L_t \) - power loss at tooth engagement.
- \( L_{ch} \) - churning power losses
L_b - bearing power losses & L_s-seal frictional power loss.

\[
L_t = W \left( \frac{0.1}{Z_1 \cos \psi} + \frac{0.3}{V + 2} \right) \text{ kW}
\]

\[
L_t = 300 \left( \frac{0.1}{29 \cos 35^\circ} + \frac{0.3}{27.3 + 2} \right) = 4.33 \text{ kW}
\]

\[
L_{ch} = c b V \left( \frac{200V \mu}{Z_1 + Z_2} \right)^{0.5} \times 10^{-3} \text{ kW}
\]

Where

V - peripheral speed (m/s)

b - face width of the gear (mm)

c - factor equal to 0.009 for splash lubrication, 0.006 for stream lubrication

\[\mu\] - Viscosity of oil at the operating temperature (cP)

\[
L_{ch} = c b V \left( \frac{200V \mu}{Z_1 + Z_2} \right)^{0.5} \times 10^{-3} \text{ kW}
\]

\[
L_{ch} = 0.006 \times 70 \times 27.3 \times \left( \frac{200 \times 27.3 \times 35}{29 + 105} \right)^{0.5} \times 10^{-3} = 0.433 \text{ kW}
\]

\[L_b = 5.23 \times 10^{-8} F \ f_b \ d \ n \ \text{ kW}\]

where

F - radial load on the bearing (N)

\[f_b\] - coefficient of friction at the bearing reduced to the shaft diameter 0.005 - 0.01 for rough estimation or refer to catalog.

d - shaft diameter (mm)

n - shaft speed (rpm)

From the catalog \[f_B = 0.002\] for roller bearings and 0.003 for ball bearings.

Bearings at A & B

\[
L_b = 5.23 \times 10^{-8} F \ f_b \ d \ n
\]

\[
= 5.33 \times 10^{-8} \times 15420 \times 0.002 \times 45 \times 2950 = 0.218 \text{ kW}
\]
Bearings at D & E

\[ L_b = 5.23 \times 10^{-8} F_f b d n \]
\[ = 5.33 \times 10^{-8} \times 15420 \times 0.003 \times 75 \times 814.92 = 0.151 \text{kW} \]
\[ L_B = 0.369 \text{kW} \]

Seal frictional power loss:

\[ L_s = T_s \omega \times 10^{-3} \text{kW} \]

Where \( T_s \) seal friction torque
\( \omega \) – angular velocity of the shaft.

**Fig. 17.23 Friction torque at various temperature for nitrile rubber S type oil seal, Shaft diameter 50mm, speed 1000 rpm**

\[ T_s = f P_r r \]

Where \( r \) = radius of the shaft [m]
\( f \) – seal friction
\( P_r \) – Radial lip load [N]

Coefficient of friction: \( f \)
\[ f = \varphi (\mu v b / P_r)^{1/3} \]
\( \varphi \) = Characteristic Number
\( \mu \) = Oil Viscosity [N.s/cm\(^2\)]
\( v \) = Linear Speed [m/s]
b = Lip Contact Width [m]

Fig. 17.23 gives the torque vs temperature chart for seal. Let the outlet oil temperature be 65°C

At 65°C, \( T_s = 0.17 \text{Nm} \) from Fig.17.23a

\[
V = \frac{\pi dn}{60000} = \frac{\pi \times 50 \times 1000}{60000} = 2.36 \text{ m/s}
\]

The operating velocity \( V = \frac{\pi \times 45 \times 2950}{60000} = 6.95 \text{ m/s} \)

\( T_s \) at operating speed of pinion shaft speed = \( 0.17 \times \left( \frac{6.95}{2.36} \right)^{1/3} = 0.244 \text{ Nm} \)

---

**Graph 17.23a Friction torque at various temperatures for nitrile rubber**

**S type oil seal, Shaft diameter 50 mm, speed 1000 rpm**

Pinion shaft seals power loss

\[
L_s = T_s \omega \times 10^{-3} = 0.244 \times 308.77 \times 2 \times 10^{-3} = 0.151 \text{ kW}
\]

Gear shaft seal power loss

\[
V = \pi \times 75 \times 814.92/60000 = 3.2 \text{ m/s}
\]

\( T_s = 0.17 \left( \frac{3.2}{2.36} \right)^{1/3} = 0.188 \text{ Nm} \)

\[
L_s = T_s \omega \times 10^{-3} = 0.188 \left( \frac{308.77}{3.62} \right) \times 2 \times 10^{-3} = 0.032 \text{ kW}
\]

Total seal friction = 0.151 + 0.032 = 0.183 kW.
Total power loss in the gearbox:
\[ L = L_t + L_{ch} + L_b + L_s \]
\[ = 4.33 + 0.433 + 0.369 + 0.183 = 5.315 \text{ kW} \]

For the operating speed of the gear 27.33 m/s, the suggested type of lubrication is oil jet lubrication.

Assuming inlet oil temperature of 40°C and outlet oil temperature of 65°C, the oil supply rate has to be:
\[ Q_e = \frac{L \times 10^3}{\rho \cdot c \cdot \Delta T} = \frac{5.315 \times 10^3}{0.88 \times 1670 \times 25} = 0.1447 \text{ lps} = 0.01447 \times 60 = 8.68 \text{ lpm} \]

Based on the details of the gearbox, the shaft details are worked out. The detailed pinion shaft drawing is shown in Fig.17.24 and that of the gear shaft in Fig.17.25. The corresponding revised dimensions of the pinion and gears are shown in Fig.17.26 and 17.27.

![Pinion shaft drawing](image)

**Fig.17.24 Pinion shaft drawing material C 45 hardened and tempered to 380 Bhn**
Fig. 17.25 Gear shaft drawing material ductile iron 120/90/02 hardened to 331 Bhn

Fig. 17.26 Pinion blank revised drawing showing all the dimensions
Fig. 17.27 Gear blank revised drawing showing all the dimensions.

Table 17.6 Details of the gearbox

<table>
<thead>
<tr>
<th>Gearbox size &amp; wt</th>
<th>745x260x1020 mm</th>
<th>330 kg</th>
<th>MS welded construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion C45 steel with</td>
<td>120/90/02 of hardness 331 Bhn</td>
<td>Shafts C-45 hardened and tempered and ground</td>
<td></td>
</tr>
<tr>
<td>hardness 380 Bhn</td>
<td>Hobbed and ground</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gear ductile iron grade</td>
<td>Hobbed and ground</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lubricant SAE 30</td>
<td>Oil jet lubrication 10 lpm</td>
<td>η = 98.2%</td>
<td></td>
</tr>
</tbody>
</table>
The gearbox is of split type with radial assembly. Gears and bearings are mounted on the shafts separately outside and assembled radially in the gearbox and the top cover is bolted in position. The oil jet and the outlet connections are made subsequently. 8 lpm oil is directed at the gear mesh and 2 lpm is directed at the bearings and seals.

The gearbox assembly views are shown in Fig. 17.28 to 17.30. The front view separately and end view are shown separately in Fig 17.28 and Fig. 17.29 for clarity. The assembly view front and side together is shown in Fig.17.30.
Fig. 17.29 End view of the gearbox

Fig. 17.30 Assembled view of the helical gearbox