Lecture – 29

Applications Linear Control Design Techniques in Aircraft Control – I

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Topics

- Brief Review of Aircraft Flight Dynamics
- An Overview of Automatic Flight Control Systems
- Automatic Flight Control Systems: Classical (Frequency Domain) Designs
Brief Review of Aircraft Flight Dynamics

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Geometry of Conventional Aircrafts

- **Horizontal Stabilizer**: Control Pitch
- **Vertical Stabilizer**: Control Yaw
- **Rudder**: Change Yaw (side to side)
- **Elevator**: Change Pitch (up and down)
- **Wing**: Generate Lift
- **Jet Engine**: Generate Thrust
- **Aileron**: Change Roll
- **Spoller**: Change Lift, Drag and Roll
- **Cockpit**: Command and Control
- **Fuselage (Body)**: Hold Things Together & Carry Payload
Basic Force Balance

- Weight
- Lift
- Drag
- Thrust
Basic Moment Balance

- Rolling
- Pitching
- Yawing
Aileron ➡ Roll
Elevator ➔ Pitch
Rudder ➡ Yaw
Airplane Dynamics: Six Degree-of-Freedom Model

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, 1995

\[
\begin{align*}
\dot{U} &= VR - WQ - g \sin \Theta + \frac{1}{m} (X + X_T) \\
\dot{V} &= WP - UR + g \sin \Phi \cos \Theta + \frac{1}{m} (Y + Y_T) \\
\dot{W} &= UQ - VP + g \cos \Phi \cos \Theta + \frac{1}{m} (Z + Z_T) \\
\dot{P} &= c_1 QR + c_2 PQ + c_3 (L + L_T) + c_4 (N + N_T) \\
\dot{Q} &= c_5 PR - c_6 (P^2 - R^2) + c_7 (M + M_T) \\
\dot{R} &= c_8 PQ - c_9 QR + c_4 (L + L_T) + c_9 (N + N_T) \\
\dot{\Phi} &= P + Q \sin \Phi \tan \Theta + R \cos \Phi \tan \Theta \\
\dot{\Theta} &= Q \cos \Phi - R \sin \Phi \\
\dot{\Psi} &= (Q \sin \Phi + R \cos \Phi) \sec \Theta \\
\dot{h} &= -\dot{\zeta}_I = U \sin \Theta - V \cos \Theta \sin \Phi - W \cos \Theta \cos \Phi
\end{align*}
\]
Representation of Longitudinal Dynamics in Small Perturbation

State space form:
\[ \dot{X} = AX + BU_c \]

\[ A = \begin{bmatrix}
X_U & X_W & 0 & -g \\
Z_U & Z_W & U_0 & 0 \\
M_U + M_w Z_U & M_w + M_w Z_W & M_Q + M_w U_0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \]

\[ B = \begin{bmatrix}
X_{\delta E} & X_{\delta T} \\
Z_{\delta E} & Z_{\delta T} \\
M_{\delta E} + M_w Z_{\delta E} & M_{\delta T} + M_w Z_{\delta T} \\
0 & 0
\end{bmatrix} \]

\[ U_c = \begin{bmatrix}
\Delta U \\
\Delta W \\
\Delta Q \\
\Delta \theta
\end{bmatrix} \]

\[ X = \begin{bmatrix}
\Delta U \\
\Delta W \\
\Delta Q \\
\Delta \theta
\end{bmatrix} \]

\[ X_U = \frac{1}{m} \left( \frac{\partial X}{\partial U} \right), \quad X_W = \frac{1}{m} \left( \frac{\partial X}{\partial W} \right) \text{ etc.} \]
Short Period Mode

- Heavily damped
- Short time period
- Constant velocity
Short Period Dynamics

State Space Equation:

\[
\begin{bmatrix}
\Delta \dot{\alpha} \\
\Delta \dot{\delta}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & 1 \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \alpha \\
\Delta \delta
\end{bmatrix} +
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix}\Delta \delta_E
\]

Transfer Function Equations:

\[
\frac{\Delta \alpha(s)}{\Delta \delta_E(s)} = \frac{A_\alpha s + B_\alpha}{A s^2 + B s + C}
\]

\[
\frac{\Delta q(s)}{\Delta \delta_E(s)} = \frac{A_q s + B_q}{A s^2 + B s + C}
\]
Long Period (Phugoid) Dynamics


- Lightly damped
- Changes in pitch attitude, altitude, velocity
- Constant angle of attack
**Long Period (Phugoid) Dynamics**

State Space Equation:

\[ \begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} a_{11} & -g \\ a_{21} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \Delta \delta_E \\ \Delta \delta_T \end{bmatrix} \]

Transfer Function Equations:

- \[ \frac{\Delta u(s)}{\Delta \delta_E(s)} = \frac{A_u s + B_u}{A s^2 + B s + C} \]
- \[ \frac{\Delta \theta(s)}{\Delta \delta_E(s)} = \frac{A_\theta s + B_\theta}{A s^2 + B s + C} \]

Assumption: \( \Delta \delta_T = 0 \)
**Representation of Lateral Dynamics in Small Perturbation**

State space form: \[ \dot{X} = AX + BU_c \]

\[
A = \begin{bmatrix}
Y_V & Y_P & -(U_0 - Y_R) & g \cos \theta_0 \\
L_V^* + \frac{I_{xz}}{I_X} N_V^* & L_P^* + \frac{I_{xz}}{I_X} N_P^* & L_R^* + \frac{I_{xz}}{I_X} N_R^* & 0 \\
N_V^* + \frac{I_{xz}}{I_Z} L_V^* & N_P^* + \frac{I_{xz}}{I_Z} L_P^* & N_R^* + \frac{I_{xz}}{I_Z} L_R^* & 0 \\
0 & 1 & 0 & 1
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 & Y_{\delta_k} \\
L_{\delta_A}^* + \frac{I_{xz}}{I_X} N_{\delta_A} & L_{\delta_R}^* + \frac{I_{xz}}{I_X} N_{\delta_R} \\
N_{\delta_A}^* + \frac{I_{xz}}{I_Z} L_{\delta_A}^* & N_{\delta_R}^* + \frac{I_{xz}}{I_Z} L_{\delta_R}^* \\
0 & 0
\end{bmatrix}
\]

\[
U_c = \begin{bmatrix}
\Delta \delta_A \\
\Delta \delta_R
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
\Delta V \\
\Delta P \\
\Delta R \\
\Delta \phi
\end{bmatrix}
\]
Lateral Dynamic Instabilities

Directional divergence
• Do not possess directional stability
• Tend towards ever-increasing angle of sideslip
• Largely controlled by rudder

Spiral divergence
• Spiral divergence tends to gradual spiraling motion & leads to high speed spiral dive
• Non–oscillatory divergent motion
• Largely controlled by ailerons
Lateral Dynamic Instabilities

Dutch roll oscillation

• Coupled directional-spiral oscillation

• Combination of rolling and yawing oscillation of same frequency but out of phase each other

• Controlled by using both ailerons and rudders
Dutch Roll Dynamics

State Space Equation:

\[
\begin{bmatrix}
\Delta \dot{\beta} \\
\Delta \dot{r}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \beta \\
\Delta r
\end{bmatrix} +
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_A \\
\Delta \delta_R
\end{bmatrix}
\]

Transfer Function Equations:

\[
\frac{\Delta \beta(s)}{\Delta \delta_A(s)} = \frac{A_\beta s + B_\beta}{As^2 + Bs + C}
\]

\[
\frac{\Delta \delta_A(s)}{\Delta \delta_R(s)} = \frac{A_r s + B_r}{As^2 + Bs + C}
\]

\[
\frac{\Delta \beta(s)}{\Delta \delta_R(s)} = \frac{\hat{A}_\beta s + \hat{B}_\beta}{As^2 + Bs + C}
\]

\[
\frac{\Delta \delta_R(s)}{\Delta \delta_R(s)} = \frac{\hat{A}_r s + \hat{B}_r}{As^2 + Bs + C}
\]
Automatic Flight Control Systems: An Overview

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“Putting it All Together”
Flight Control System

Aircraft Sensors
- Position Gyros
- Rate Gyros
- etc.
- Orientation
- Velocity
- Altitude
- etc.

Sensor Measurements

Flight Control Computer

Controller Commands
- Throttle Position
- Rudder Position
- Elevator Position
- Aileron Position
- etc.

Pilot Commands
- Flight Path Command
- Velocity Command
- Altitude Command
- etc.

Aircraft Cockpit

Aircraft Control Effectors
- Ailerons
- Rudder
- Elevator
- Engines
Sensors

- **Altimeter**: Height above sea level
- **Air Data System**: Airspeed, Angle of Attack, Mach No., Air Temperature etc.
- **Magnetometer**: Heading
- **Inertial Navigation System (INS)**
  - **Accelerometers**: Translational motion of the aircraft in the three axes
  - **Gyroscopes**: Rotational motion of the aircraft in the three axes
- **GPS**: Accurate position, ground speed

*The transfer function for most sensors can be approximated by a gain* $k$
Actuators

- Electrical actuators:
  - It's a second order system in general
  - It can be approximated to a first order system with small angle displacements

- Hydraulic / Pneumatic actuators
  - First order system

- Combination of the above
Applications of Automatic Flight Control Systems

- **Cruise Control Systems**
  - Attitude control (to maintain pitch, roll and heading)
  - Altitude hold (to maintain a desired altitude)
  - Speed control (to maintain constant speed or Mach no.)

- **Stability Augmentation Systems**
  - Stability enhancement
  - Handling quality enhancement

- **Landing Aids**
  - Alignment control (to align wrt. runway centre line)
  - Glideslope control
  - Flare control
Techniques for Autopilot Design

- Frequency domain techniques:
  - Root locus
  - Bode plot
  - Nyquist plot
  - PID design etc.

- Time domain techniques:
  - Pole placement design
  - Lyapunov design
  - Optimal control design etc.
Automatic Flight Control Systems:
Frequency Domain Designs

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Velocity Hold Control System

- The forward speed of an airplane can be controlled by changing the thrust produced by propulsion system.

- The function of the speed control system is to maintain the some desired flight speed.

![Diagram of Velocity Hold Control System]
Attitude Control System

- Sense the angular position
- Compare the angular position with the desired angular position
- Generate commands proportional to the error signal
Roll Attitude Autopilot


Components:
- Model for roll dynamics (transfer function)
- Attitude gyro
- Comparator
- Aileron actuator

Reference:
Roll Attitude Autopilot

Problem:

Design a roll attitude control system to maintain a wings level attitude for a vehicle having the following characteristics.

\[ L_p = -0.5 \text{ rad} / \text{s} \quad L_{\delta_a} = 2.0 / \text{s}^2 \]

The system performance is to have damping ratio \( \zeta = 0.707 \) and an undamped natural frequency \( \omega_n = 10 \text{ rad/s} \).

Assume the aileron actuator and the sensor (gyro) can be represented by the gains \( k_a \) and \( k_s \).
Roll Attitude Autopilot


The system transfer function
\[
\frac{\Delta \phi(s)}{\Delta \delta_a(s)} = \frac{L_{\delta a}}{s(s - L_p)}
\]

Forward path transfer function
\[
G(s) = \frac{\Delta \phi(s)}{e(s)} \cdot \frac{\Delta \delta_a(s)}{\Delta \delta_a(s)} = k_a \frac{L_{\delta a}}{s(s - L_p)}
\]

\[
H(s) = k_s = 1 \quad \text{(unity feedback assumption)}
\]

The loop transfer function
\[
G(s)H(s) = \frac{k}{s(s + 0.5)}, \quad \text{where} \quad k = k_a L_{\delta a}
\]
Roll Attitude Autopilot

The desired damping needed is $\zeta = 0.707$

We know $\zeta = \cos \theta$. Hence,
draw a line of $45^0$ from the origin.
Any root intersecting this line will have $\zeta = 0.707$.

Gain is determined from:
\[
\frac{|k|}{|s||s + 0.5|} = 1,
\]
This leads to $k = 0.0139$. However,
$\omega_n = 0.35 \text{ rad} / \text{s}$ (much lower than desired!)
Roll Attitude Autopilot

Moving pole $P_1$ to $P_2$ is impractical, unless there is an increase in wingspan of the aircraft..!
Hence, the cure is to have a stability augmentation system.
Roll Attitude Autopilot


- Compensator is added in form of rate feedback loop to meet desired damping and natural frequency.
- The inner loop transfer function can be expressed as follows:
  \[
  \frac{\Delta p(s)}{\Delta \delta_a(s)} = \frac{L_{\delta a}}{(s - L_p)}
  \]
- Transfer function of inner loop
  \[
  G(s)_{IL} = \frac{k_{IL}}{s + 0.5}, \quad H(s)_{IL} = 1(k_{rg}), \quad k_{IL} = k_{as}L_{\delta a}
  \]

\[
M(s)_{IL} = \frac{G(s)_{IL}}{1 + G(s)_{IL}H(s)_{IL}} = \frac{k_{IL}}{s + 0.5 + k_{IL}}
\]
Roll Attitude Autopilot

Inner loop gain is selected to move the augmented root farther on the negative real axis.

If the inner loop is located at $s = -14.14$, then the root locus will shift to the left and desired $\xi$ and $\omega_n$ can be achieved.
Stability Augmentation System (SAS)

- Inherent stability of an airplane depends on the aerodynamic stability derivatives.
- Magnitude of derivatives affects both damping and frequency of the longitudinal and lateral motion of an airplane.
- Derivatives are function of the flying characteristics which change during the entire flight envelope.
- Control systems which provide artificial stability to an airplane having undesirable flying characteristics are commonly called as **stability augmentation systems**.
Example: SAS

Consider an aircraft with poor short period dynamic characteristics. Assume one degree of freedom (only pitching motion about CG) to demonstrate the SAS.

Short period dynamics:

\[ \ddot{\theta} - (M_q + M_\alpha) \dot{\theta} + M_\alpha \theta = M_\delta \delta \]

Substituting the numerical values

\[ \ddot{\theta} - 0.071 \dot{\theta} + 5.49 \theta = -6.71 \delta_e \]

This leads to \( \zeta_{sp} = 0.015 \quad \omega_{nsp} = 2.34 \text{ rad} / \text{s} \)

It is seen that the airplane has poor damping (flying quality).
Example: SAS

One way to improve damping is to provide rate feedback. Artificial damping is provided by producing an elevator deflection in proportion to pitch rate and then adding it to the pilot's input; i.e.

\[
\delta_e = \delta_{ep} + k\dot{\theta}^{\text{Artificial command}}
\]

Rate gyro measures the \( \dot{\theta} \) and creates an electrical signal to provide \( k\dot{\theta} \) in addition to \( \delta_e \).

With this, the modified dynamics becomes:

\[
\dot{\theta} - (0.071 + 6.71k)\dot{\theta} + 5.49\theta = -6.71\delta_e
\]

\[
2\zeta\omega_n = 0.071 + 6.71k, \quad \omega_n^2 = 5.49
\]

Hence, by varying \( k \), the desired damping is achieved.
Landing System


Key Components:

• Alignment control (to align wrt. runway centre line)

• Glideslope control

• Flare control
Alignment Control Through Electronic Aid and Pilot Input

Glide Slope: Pitch Control
Glide Slope: Speed Control

##### Flare: Sink Rate Control

Thanks for the Attention...!