In this lecture...

• Performance parameters: cascade analysis
• 2-D losses in axial compressor stage – primary losses
Performance parameters

• Measurements from cascade: velocities, pressures, flow angles …
• Loss in total pressure expressed as total pressure loss coefficient

\[
\bar{\omega}_{PLC} = \frac{P_{01} - P_{02}}{\frac{1}{2} \rho V_1^2}
\]

• Total pressure loss is very sensitive to changes in the incidence angle.
• At very high incidences, flow is likely to separate from the blade surfaces, eventually leading to stalling of the blade.
Performance parameters

• Blade performance/loading can be assessed using static pressure coefficient:

\[ C_P = \frac{P_{local} - P_{ref}}{\frac{1}{2} \rho V_1^2} \]

Where, \( P_{local} \) is the blade surface static pressure and \( P_{ref} \) is the reference static pressure (usually measured at the cascade inlet).

• The \( C_P \) distribution (usually plotted as \( C_P \) vs. \( x/C \)) gives an idea about the chordwise load distribution.
Performance parameters

- Deflection, degrees
- Total pressure loss coefficient
- Position along cascade
- Location of the blade trailing edge
Performance parameters

(a) Normal operation  (b) Stalled operation

Stalled or separated flow
Performance parameters

Incidence angle, degrees

Total pressure loss coefficient
Losses in a compressor blade

• Nature of losses in an axial compressor
  – Viscous losses
  – 3-D effects like tip leakage flows, secondary flows etc.
  – Shock losses
  – Mixing losses

• Estimating the losses crucial designing loss control mechanisms.

• However isolating these losses not easy and often done through empirical correlations.

• Total losses in a compressor is the sum of the above losses.
Losses in a compressor blade

• Viscous losses
  – Profile losses: on account of the profile or nature of the airfoil cross-sections
  – Annulus losses: growth of boundary layer along the axis
  – Endwall losses: boundary layer effects in the corner (junction between the blade surface and the casing/hub)

• 3-D effects:
  – Secondary flows: flow through curved blade passages
  – Tip leakage flows: flow from pressure surface to suction surface at the blade tip
Losses in a compressor blade

- The loss manifests itself in the form of stagnation pressure loss (or entropy increase).

\[
\frac{\Delta s}{R} = -\ln \frac{P_{02}}{P_{01}} = -\ln\left[1 - \frac{(\Delta P_0)_{\text{loss}}}{P_{01}}\right]
\]

Expanding the above equation in an infinite series,

\[
\frac{\Delta s}{R} = \frac{(\Delta P_0)_{\text{loss}}}{P_{01}} + \frac{1}{2}\left(\frac{(\Delta P_0)_{\text{loss}}}{P_{01}}\right)^2 + \ldots
\]

Neglecting higher order terms, \(\frac{\Delta s}{R} = \frac{(\Delta P_0)_{\text{loss}}}{P_{01}}\)

Since, \(\omega = \frac{(\Delta P_0)_{\text{loss}}}{\frac{1}{2} \rho V_1^2} = \frac{\Delta s}{R} \frac{P_{01}}{\frac{1}{2} \rho V_1^2}\)

or, \(\frac{\Delta s}{R} = \left(\frac{\omega \rho V_1^2}{2P_{01}}\right)\)
Losses in a compressor blade

- The overall losses in a turbomachinery can be summarised as:

\[ \omega = \omega_p + \omega_{sh} + \omega_s + \omega_L + \omega_E \]

Where, \( \omega_p \): profile losses

\( \omega_{sh} \): shock losses

\( \omega_s \): secondary flow loss

\( \omega_L \): tip leakage loss

\( \omega_E \): Endwall losses
2-D Losses in a compressor blade

• 2-D losses are relevant only to axial flow turbomachines.

• These are mainly associated with blade boundary layers, shock-boundary layer interactions, separated flows and wakes.

• The mixing of the wake downstream produces additional losses called mixing losses.

• The maximum losses occur near the blade surface and minimum loss occurs near the edge of the boundary layer.
2-D Losses in a compressor blade

- 2-D losses can be classified as:
  - Profile loss due to boundary layer, including laminar and/or turbulent separation.
  - Wake mixing losses
  - Shock losses
  - Trailing edge loss due to the blade.
The profile loss depends upon:

- Flow parameters like Reynolds number, Mach number, longitudinal curvature of the blade, inlet turbulence, free-stream unsteadiness and the resulting unsteady boundary layers, pressure gradient, and shock strength.
- Blade parameters like: thickness, camber, solidity, sweep, skewness of the blade, stagger angle and blade roughness.
2-D Losses in a compressor blade

- The mixing losses arise as a result of the mixing of the wake with the freestream.
- This depends upon, in addition to the parameters mentioned in the previous slide, the distance downstream.
- The physical mechanism is the exchange of momentum and energy between the wake and the freestream.
- This transfer of energy results in the decay of the free shear layer, increased wake centre line velocity and increased wake width.
2-D Losses in a compressor blade

• At far downstream, the flow becomes uniform.
• Theoretically, the difference between the stagnation pressure far downstream and the trailing edge represents the mixing loss.
• Most loss correlations are based on measurements downstream of the trailing edge (1/2 to 1 chord length) and therefore do not include all the mixing losses.
• If there is flow separation, the losses would include losses due to this zone and at its eventual mixing downstream.
2-D Losses in a compressor blade

The profile and mixing losses along a streamline can be written as:

$$\bar{\omega}_{p+m} = \frac{2(P_{0t} - P_{02})}{\rho V_1^2}$$

To determine the above, it is necessary to relate the static pressure difference and velocities to the displacement and momentum thickness of the blade boundary layer at the trailing edge.
2-D Losses in a compressor blade

Detailed derivation of these correlations are given in Lakshminarayana's book (Chapter 6).

\[ \bar{\omega}_{p+m} = \frac{2(P_{0t} - P_{02})}{\rho V_1^2} = \frac{2(p_t - p_2)}{\rho V_1^2} + \frac{V_t^2 - V_2^2}{V_1^2} \]

This is further expressed as:

\[ \bar{\omega}_{p+m} \sec^2 \alpha_1 = \left[ \frac{2\Theta + \Delta^2}{(1 - \Delta)^2} + \tan^2 \alpha_2 \left\{ \frac{(1 - \Delta)^2}{(1 - \Theta - \Delta)^2} - 1 \right\} \right] \]

Neglecting higher order terms,

\[ \bar{\omega}_{p+m} \sec^2 \alpha_1 = 2(\Theta + \Theta \tan^2 \alpha_2) \]

Where, \( \Delta \) is the blockage (related to displacement thickness) and \( \Theta \) is the momentum thickness.
2-D Losses in a compressor blade

• Thus, in a simplified manner, we see that the profile loss can be estimated based on the momentum thickness.
• The above loss correlation includes both profile and wake mixing loss.
• If flow separation occurs, additional losses are incurred. This is because the pressure distribution is drastically altered beyond the separation point.
• The losses increase due to increase in boundary layer displacement and momentum thicknesses.
2-D Losses in a compressor blade

- In addition to the losses discussed above, boundary layer growth and subsequent decay of the wake causes deviation in the outlet angle.
- An estimate of this is given as:
  \[ \tan \alpha_2 \approx (1 - \Theta - \Delta) \tan \alpha_t \]
- Hence, viscous effect in a turbomachine always leads to decrease in the turning angle.
- The values of displacement and momentum thicknesses, depend upon, variation of freestream velocity, Mach number, skin friction, pressure gradient, turbulence intensity and Reynolds number.
2-D Losses in a compressor blade

• In general, the loss estimation may be carried out using one of the following methods:
  • Separate calculation of the potential or inviscid flow and the displacement and momentum thicknesses. Subsequently, use the equation discussed previously.
  • Using a Navier-Stokes based computational code. Here the local and the integrated losses can be computed directly.
Mach number and shock losses

- The static pressure rise in a compressor increases with Mach number.
- Thus the pressure gradient increases with increase in Mach number.
- This means that the momentum thickness and hence the losses increase with Mach number.
- Increasing Mach numbers also lead to increase in shock losses.
- At transonic speeds, the shock losses are very sensitive to leading and trailing edge geometries.
Mach number and shock losses

• An estimate of the 2-D shock losses for a compressor must include:
  • The losses due to the leading edge bluntness with supersonic upstream Mach number.
  • The location of the passage shock can be determined from inviscid theories. If the shock strength is known, the losses can be estimated.
  • The losses due to boundary layer growth and the shock-boundary layer interaction are most difficult to estimate. The contribution however is small for weak shocks.
Mach number and shock losses

- One of the empirical correlations for the shock loss was given by Freeman and Cumpsty (1989).

\[
\omega_{sh} = \frac{(\Delta P_0)_{\text{loss}}}{P_{01} - p_1} = \left[ \frac{(\Delta P_0)_{\text{loss}}}{P_{01} - p_1} \right]_{\text{normal shock}} 
+ \left[ 2.6 + 0.18(\alpha'_1 - 65^0) \right] 10^{-2} (\alpha_1 - \alpha'_1)
\]

where, \(\alpha'_1\) is the blade inlet angle.

- This is valid for an incidence angle upto 5\(^\circ\).
- These empirical correlations are however, derived using the 2-D assumption.
- Actual flows are seldom 2-D in nature.
In this lecture...

• Performance parameters: cascade analysis
• 2-D losses in axial compressor stage – primary losses
In the next lecture...

- Tutorial: solved examples and tutorial problems.