TURBOMACHINERY
AERODYNAMICS

Lect 26

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Tutorial - 4
Solved Problems
And
Exercise Problems
On
3-D flows in Axial Flow Turbine
Example 1.

Following data apply to a constant nozzle exit angle ($\alpha_2$) axial turbine design:
Temp. drop, $\Delta T = 150$ K; at hub $U_{2h} = 300$ m/s; at tip $U_{2t} = 400$ m/s; $\alpha_2 = 60$; $\alpha_3 = 0$; and Radius ratio given is, $r_h / r_t = 0.75$

(a) Complete the design velocity diagrams at hub, mean and tip of the stage
(b) Calculate the velocity components if the design is free vortex for the turbine and compare the values with (a)
Solution 1:

At the rotor inlet station we know,

\[
\frac{C_{w2}}{C_{w2m}} = \frac{C_{a2}}{C_{a2m}} = \frac{C_2}{C_{2m}} = \left(\frac{r}{r_m}\right)^{\sin^2 \alpha_2}
\]

And, at the rotor exit

\[
C_{a3}^2 = C_{a3m}^2 + 2U_mC_{w2m}\left[1 - \left(\frac{r}{r_m}\right)^{\cos^2 \alpha_2}\right]
\]

and

\[
r_m/r_t = 0.875, \text{ and } r_m/r_h = 1.166
\]
Work done by the rotor is given by (for $\alpha_3 = 0$)

$$U \left( C_{w2} + C_{w3} \right) = \Delta H_0 = c_p \Delta T = U_m C_{w2m}$$

From which we can write $C_{w2m} = 492$ m/s

$$C_{a2m} = C_{w2m} \cot \alpha_2 = 284 \text{ m/s} = C_{a3m}$$

At the rotor hub inlet

$$C_{a2h} = C_{a2m} \left( \frac{r_m}{r} \right)^{\sin^2 \varphi} = 318.8 \text{ m/s}$$

$$C_{w2h} = C_{w2m} \left( \frac{r_m}{r} \right)^{\sin^2 \varphi} = 552.2 \text{ m/s}$$
At the rotor tip inlet

\[ C_{a2t} = C_{a2m} \left( \frac{r_m}{r} \right)^{sin^2 \frac{\alpha}{2}} \frac{\varphi}{2} = 257 \text{ m/s} \]

\[ C_{w2t} = C_{w2m} \left( \frac{r_m}{r} \right)^{sin^2 \frac{\alpha}{2}} \frac{\varphi}{2} = 447 \text{ m/s} \]
At the rotor tip outlet

\[ C_{a3} = \sqrt{C_{a3m}^2 + 2U_m C_{w2m} \left[ 1 - \left( \frac{r}{r_m} \right)^2 \cos^2 \alpha_2 \right]} \]

From which we can calculate the axial velocities,

- \( C_{a3t} = 262 \text{ m/s} \)
- \( C_{a3h} = 306 \text{ m/s} \)
- \( C_{w3} \) is constant radially
(b) Free vortex stage design and comparison
For Free vortex design we have established in the last lecture

\[ C_{w3}\cdot r = \text{constant}, \ C_{a3} = \text{const} = C_{a2} \]

<table>
<thead>
<tr>
<th></th>
<th>Constant Nozzle</th>
<th>Free Vortex</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{a2h} )</td>
<td>318.8</td>
<td>284</td>
</tr>
<tr>
<td>( C_{a2m} )</td>
<td>284</td>
<td>284</td>
</tr>
<tr>
<td>( C_{a2t} )</td>
<td>257</td>
<td>284</td>
</tr>
<tr>
<td>( C_{w2h} )</td>
<td>552</td>
<td>574</td>
</tr>
<tr>
<td>( C_{w2m} )</td>
<td>492</td>
<td>492</td>
</tr>
<tr>
<td>( C_{w2m} )</td>
<td>447</td>
<td>430</td>
</tr>
<tr>
<td>( C_{a3h} )</td>
<td>306</td>
<td>284</td>
</tr>
<tr>
<td>( C_{a3m} )</td>
<td>284</td>
<td>284</td>
</tr>
<tr>
<td>( C_{a3t} )</td>
<td>262.6</td>
<td>284</td>
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</tbody>
</table>
Example 2
It is proposed that for design of an axial flow turbine two design methods are to be explored:

A) \( C_{w2m} = C_{w2h} = C_{w2t} \)

B) \( C_{a2t} = C_{a2h} \left( \frac{r_h}{r_t} \right)^{\sin^2 \alpha_2} \)

and,

c) \( C_{w2t}/ C_{w2h} = \frac{r_h}{r_t} \)

Common design data prescribed are: \( C_{am} = 200 \text{ m/s} \); \( \alpha_2 = 60 \); \( \alpha_3 = 0 \); \( R_x = 0.5 \); and \( r_h/r_t = 0.8 \)

Complete the velocity diagrams for all the cases.
Solution 2:

From the prescribed data:
One can calculate that: \( \frac{r_m}{r_t} = 0.889; \frac{r_t}{r_m} = 1.11 \)

\( C_{w2m} = C_{a2m} \times \tan \alpha_2 = 346.5 \text{ m/s} \); and \( C_{w3m} = 0 \)

For all the cases, \( R_x = 0.5 \) is prescribed at mean
Hence, from symmetrical blading concept
\( \alpha_{2m} = \beta_{3m} = 60^0 \); \( \alpha_{3m} = \beta_{2m} = 0^0 \)

Also, \( U_m = C_{w2m} = 346.5 \text{ m/s} \) and hence at any radius, \( U_h = 308 \text{ m/s} \); \( U_t = 385 \text{ m/s} \)
For Case (A)

This is a fluid behaving like a ‘solid body’ case for which \( n = 0 \) in the equation \( C_w = r^n \)

The axial speed is calculated from the axial velocity expression derived from the energy equation for the case \( n=0 \)

\[
C_{a2} = C_{a2m} \sqrt{1 - 2 \tan^2 \alpha_{2m} \ln \left( \frac{r}{r_m} \right)}
\]

All the angles across the rotor may be also calculated from above
Tabulated results of Case A

<table>
<thead>
<tr>
<th></th>
<th>$C_{a_2}$</th>
<th>$C_{a_3}$</th>
<th>$C_{w_2}$</th>
<th>$C_{w_3}$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
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<tbody>
<tr>
<td>Hub</td>
<td>261.3</td>
<td>200</td>
<td>346.5</td>
<td>0</td>
<td>53</td>
<td>0</td>
<td>8.4</td>
<td>57</td>
</tr>
<tr>
<td>Mean</td>
<td>200</td>
<td>200</td>
<td>346.5</td>
<td>0</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>Tip</td>
<td>121.3</td>
<td>200</td>
<td>346.5</td>
<td>0</td>
<td>70.7</td>
<td>0</td>
<td>-17</td>
<td>62.5</td>
</tr>
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</table>
Case (B)

Prescribed condition is $C_{a2t} = C_{a2h} \left( \frac{r_h}{r_t} \right)^{\sin^2 \varphi} \frac{a_{2t}}{a_{2h}}$

Which essentially means: $\frac{C_{a2t}}{C_{a2h}} = \frac{C_{a2t}}{C_{a2m}} = \frac{C_{a2m}}{C_{2h}}$

For constant nozzle angle: $C_{a2} = C_{a2m} \left( \frac{r_m}{r} \right)^{\sin^2 \varphi} \frac{a_{2}}{a_{2m}}$

$C_{w2} = C_{w2m} \left( \frac{r_m}{r} \right)^{\sin^2 \varphi} \frac{a_{2}}{a_{2m}}$; $C_{2} = C_{2m} \left( \frac{r_m}{r} \right)^{\sin^2 \varphi} \frac{a_{2}}{a_{2m}}$
At station 3, exit of the rotor,

\[ \alpha_3 = 0 \ ; \ C_{w3} = 0 \]

And the expression for axial velocity is

\[
C_{a3}^2 = C_{a3m}^2 + 2U_mC_{w2m}\left[1 - \left(\frac{r}{r_m}\right)\cos^2\alpha_2\right]
\]
### Tabulated results of Case B

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<tr>
<th></th>
<th>$C_{a_2}$</th>
<th>$C_{a_3}$</th>
<th>$C_{w_2}$</th>
<th>$C_{w_3}$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
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<tbody>
<tr>
<td>Hub</td>
<td>218.5</td>
<td>216.7</td>
<td>378.5</td>
<td>0</td>
<td>60</td>
<td>0</td>
<td>17.9</td>
<td>54</td>
</tr>
<tr>
<td>Mean</td>
<td>200</td>
<td>200</td>
<td>346.5</td>
<td>0</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>Tip</td>
<td>185</td>
<td>183.3</td>
<td>320</td>
<td>0</td>
<td>60</td>
<td>0</td>
<td>-19.4</td>
<td>69</td>
</tr>
</tbody>
</table>
For Case (C)

Since $C_{w2t}/C_{w2h} = r_h/r_t$ – this is Free Vortex law

Same may be applied at rotor outlet also:

$C_{a2} = \text{const} = C_{a3}$ at mean radius

The results are summarized in the table:
## Tabulated results of Case C

<table>
<thead>
<tr>
<th></th>
<th>$C_{a2}$</th>
<th>$C_{a3}$</th>
<th>$C_{w2}$</th>
<th>$C_{w3}$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\beta_2$</th>
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</thead>
<tbody>
<tr>
<td>Hub</td>
<td>200</td>
<td>200</td>
<td>389.7</td>
<td>0</td>
<td>62.8</td>
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<td>22.25</td>
<td>57</td>
</tr>
<tr>
<td>Mean</td>
<td>200</td>
<td>200</td>
<td>346.5</td>
<td>0</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>Tip</td>
<td>200</td>
<td>200</td>
<td>311.8</td>
<td>0</td>
<td>57.3</td>
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<td>-20.8</td>
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### All Three cases compared: Design velocity diagrams

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<th>$C_{a3}$</th>
<th>$C_{w2}$</th>
<th>$C_{w3}$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
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<tr>
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<td>A</td>
<td>261.3</td>
<td>200</td>
<td>346.5</td>
<td>0</td>
<td>53</td>
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<td>8.4</td>
<td>57</td>
</tr>
<tr>
<td>Hub</td>
<td>B</td>
<td>218.5</td>
<td>216.7</td>
<td>378.5</td>
<td>0</td>
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Next Lecture ------

Turbine blade cooling