Module 2
Lecture 10
Permeability and Seepage -6

Topics

1.2.11 Plotting of Phreatic Line for Seepage through Earth Dams
1.2.12 Entrance, Discharge, and Transfer Conditions of Line of Seepage through Earth Dams
1.2.13 Flow net Construction for Earth Dams
1.2.14 Filter Design

1.2.11 Plotting of Phreatic Line for Seepage through Earth Dams

For construction of flow nets for seepage through earth dams, the phreatic line needs to be established first. This is usually done by the method proposed by Casagrande (1937) and is shown in Figure 2.60a. Note that \( aefb \) in Figure 2.60a is the actual phreatic line. The curve \( a'efb'c' \) is a parabola with its focus at \( c \); the phreatic line coincides with this parabola, but with some deviations at the upstream and the downstream force. At a point \( a \), the phreatic line starts at an angle of \( 90^\circ \) to the upstream face of the dam and \( aa' = 0.3\Delta \).

![Diagram of phreatic line](image)

**Figure 2.60** Determination of phreatic line for seepage through an earth dam
The parabola $a'eb'c'$ can be constructed as follows:

1. Let the distance $cc'$ be equal to $p$. Now referring to Figure 2.60b, $Ac = AD$ (based on the properties of a parabola), $Ac = \sqrt{x^2 + z^2}$, and $AD = 2p + x$, thus
   \[ \sqrt{x^2 + z^2} = 20 + x \]  
   At $x = d, z = H$. substituting these conditions into equation (2.196) and rearranging, we obtain
   \[ p = \frac{1}{2}(\sqrt{d^2 + H^2} - d) \]  
   Since $d$ and $H$ are known, the value of $p$ can be calculated.

2. From equation (2.196),
   \[ x^2 + z^2 = 4p^2 + x^2 + 4px \]
   \[ x = \frac{z^2 - 4p^2}{4p} \]  
   With $p$ known, the values of $x$ for various values of $z$ can be calculated from equation (2.198) and the parabola can be constructed.

To complete the phreatic line, the portion $ae$ has to be approximated and drawn by hand. When $\beta < 30^\circ$, the value of $l$ can be calculated from equation (2.183) as

\[ l = \frac{d}{\cos \beta} - \sqrt{\frac{d^2}{\cos^2 \beta} - \frac{H^2}{\sin^2 \beta}} \]

Note that $l = bc$ in Figure 2.60a. Once point $b$ has been located, the curve $fb$ can be approximately drawn by hand.

**Figure 2.61** Plot of $\Delta l / (1 + \Delta l)$ against downstream slope angle. *(After A. Casagrande, *Seepage through Dams. Contribution to Soil Mechanics, 1925-1940, *Boston Society of Civil Engineering, Boston, 1937.)*
If $\beta > 30^\circ$, Casagrande proposed that the value of $l$ can be determined by using the graph given in Figure 2.61. In Figure 2.60a, $b'b = 1, and \ bc = l$. After locating the point $b$ on the downstream face, the curve $fb$ can be approximately drawn by hand.

**Example 1.10.** An earth dam section is shown in Figure 2.62. Plot the phreatic line for seepage. For the earth dam section, $k_x = k_z$.

**Solution**

\[
\beta = \tan^{-1}(1/1.5) = 33.60^\circ
\]

\[
\Delta = 70 \cot 45^\circ = 70 \text{ ft}
\]

\[
aa' = 0.3\Delta = 0.3(70) \text{ ft}
\]

And \( d = 80 \cot 33.69^\circ + 15 + 10 \cot 45^\circ + 21 = 120 + 15 + 10 + 21 = 166 \text{ ft} \)

From equation (2.197),

\[
p = \frac{1}{2} \left( \sqrt{a^2 + h^2} - d \right) = \frac{1}{2} \left( \sqrt{166^2 + 70^2} - 166 \right) \text{ ft}
\]

\[
= \frac{1}{2} \left( 180.16 - 166 \right) = 7.08 \text{ ft}
\]

Using equation (2.198), we can now determine the coordinates of several points of the parabola $\text{a'efb'c'}$:

<table>
<thead>
<tr>
<th>$z, \text{ft}$</th>
<th>$x$ from equation, $\text{ft}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>166</td>
</tr>
<tr>
<td>65</td>
<td>142.1</td>
</tr>
<tr>
<td>60</td>
<td>120.04</td>
</tr>
<tr>
<td>55</td>
<td>99.73</td>
</tr>
<tr>
<td>50</td>
<td>81.2</td>
</tr>
<tr>
<td>45</td>
<td>64.42</td>
</tr>
</tbody>
</table>

Using the values of $x$ and corresponding $z$ calculated in the above table, the basic parabola has been plotted in Figure 2.62.
We calculate \( l \) as follows. The equation of the line \( cb' \) can be given by \( z = x \tan \beta \), and the equation of the parabola [equation (2.198)] is \( x = (z^2 - 4p^2)/4p \). The coordinates of point \( b' \) can be determined by solving the above two equations:

\[
x = \frac{z^2 - 4p^2}{4p} = \frac{(x \tan \beta)^2 - 4p^2}{4p}
\]

Or \( x^2 \tan^2 \beta - 4px - 4p^2 = 0 \)

Hence

\[
x^2 \tan^2 33.69^\circ - 4(7.08)x - 4(7.08)^2 = 0
\]

\[
0.44x^2 - 28.32x - 200.5 = 0
\]

The solution of the above equation gives \( x = 70.22 \) ft. so

\[
cb' = \sqrt{70.22^2 + (70.22 \tan 33.69^\circ)^2} = 84.39 \text{ ft} = l + \Delta l
\]

From Figure 2.61, for \( \beta = 33.69^\circ \),

\[
\frac{\Delta l}{l + \Delta l} = 0.366 \quad \Delta l = (0.366)(84.39) = 30.9 \text{ ft}
\]

\[
l = (l + \Delta l) - (\Delta l) = 84.39 - 30.9 = 53.49 \text{ ft}
\]

So, \( l = cb = 54 \) ft.

The curve portions \( ae \) and \( fb \) can now be approximately drawn by hand which completes the phreatic line \( aefb \) Figure 2.62.
1.2.12 Entrance, Discharge, and Transfer Conditions of Line of Seepage through Earth Dams

A. Casagrande (1937) analyzed the entrance, discharge, and transfer conditions for the line of seepage through earth dams. When we consider the flow from a free draining material (coefficient of permeability very large; \( k_1 \approx \infty \)) into a material of permeability \( k_2 \), it is called an entrance. Similarly, when the flow is from a material of permeability \( k_1 \) into a free draining material (\( k_2 \approx \infty \)) it is referred to as discharge. **Figure 2.63** shows various entrances, discharge, and transfer conditions. The transfer conditions show the nature of deflection of the line of seepage when passing from a materials of permeability \( k_2 \).

Using the conditions given in Figure 2.63 we can determine the nature of the phreatic lines for various types of earth dam sections.

1.2.13 Flow net Construction for Earth Dams

With knowledge of the nature of the phreatic line and the entrance, discharge, and transfer conditions, we can proceed to draw flow nets for earth dam sections. **Figure 2.64** shows an earth
dam section that is homogeneous with respect to permeability. To draw the flow net, the following steps must be followed:

![Flow-net constructions for an earth dam.](image)

**Figure 2.64** Flow-net constructions for an earth dam.

1. Draw the phreatic line, since this is known.
2. Note that $ag$ is an equipotential line and that $gc$ is a flow line.
3. It is important to realize that the pressure head at any point on the phreatic line is zero; hence the difference of total head between any two equipotential lines should be equal to the difference in elevation between the points where these equipotential lines intersect the phreatic line.
   Since loss of hydraulic head between any two consecutive equipotential lines is the same, determine the number of equipotential drops, $N_d$, the flow net needs to have and calculate $\Delta h = h/N_d$.
4. Draw the head lines for the cross section of the dam. The points of intersection of the head lines and the phreatic lines are the points from which the equipotential lines should start.
5. Draw the flow net, keeping in mind that the equipotential lines and flow lines must intersect at right angles.
6. The rate of seepage through the earth dam can be calculated from the relation given in equation (2.118), $q = k(h(N_f/N_d))$.

In Figure 2.64, the number of flow channels, $N_f$, is equal to 2.3. The top two flow channels have square flow elements, and the bottom flow channel has elements with a width-to-length ratio of 0.3. Also, $N_d$ in Figure 2.64 is equal to 10.

If the dam section is anisotropic with respect to permeability, a transformed section should first be prepared in the manner outlines in section 2.2.5. The flow net can then be drawn on the transformed section, and the rate of seepage obtained from equation (2.124).

If the phreatic line for the dam section is not known, a trial and error procedure will have to be adopted for the construction of flow nets (Cedergren, 1977). This technique is demonstrated in Figure 2.65. The steps to obtain the flow net are as follows:

1. Draw the head lines on the cross section of the dam. Also draw the approximate zone of the phreatic line as shown in Figure 2.65a.
2. Assume a trial saturation line (phreatic line) \(ab\) as shown in Figure 2.65b. Draw an approximate flow net. Now check the number of flow channels between any two consecutive equipotential lines. If the flow net is correctly drawn, the number of flow channels between any two consecutive equipotential lines should be the same. If not, some adjustment has to be made by moving the phreatic line and the flow and equipotential lines. (The arrows in Figure 2.65b show the direction of needed correction.)

3. After a few trials, the final flow net can be obtained as shown in Figure 2.65c \((N_f = 1.2\) and \(N_d = 5\)

![Flow-net construction for an earth dam section with unknown phreatic line.](image)

Figure 2.65 Flow-net construction for an earth dam section with unknown phreatic line. (Modified after H. R. Cedergren, “Seepage, Drainage and Flow Nets,” 2d ed., Wiley, New York, 1977)

Figure 2.66 and 1.67 shows some typical flow nets through earth dam sections.

A flow net for seepage through a zoned earth dam section is shown in Figure 2.68. The soil for the upstream half of the dam has permeability \(k_1\), and the soil for the downstream half of the dam has permeability \(k_2 = 5k_1\). The phreatic line has to be plotted by trial and error. As shown in Figure 2.37b, here the seepage is from a soil of low permeability (upstream half) to a soil of high permeability (downstream half). From equation (2.125),

\[
\frac{k_1}{k_2} = \frac{b_2/l_2}{b_1/l_1}
\]
If \( b_1 = l_1 \) and \( k_2 = 5k_1 \), \( b_2/l_2 = 1/5 \). For that reason, square flow elements have been plotted in the upstream half of the dam, and the flow elements in the downstream half have a width-to-length ratio of 1/5. The rate of seepage can be calculated by using the following equation:

\[
q = k_1 \frac{h}{N_d} N_{f(1)} = k_2 \frac{h}{N_d} N_{f(2)}
\]

Where \( N_{f(1)} \) is the number of full flow channels in the soil having permeability \( k_1 \), and \( N_{f(2)} \) is the number of full flow channels in the soil having a permeability \( k_2 \).

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**Figure 2.66** Typical flow net for an earth dam with rock toe filter

**Figure 2.67** Typical flow net for an earth dam with chimney drain
1.2.14 Filter Design

When seepage water flows from a soil with relatively fine grains into a coarser material, there is a danger that the fine soil particles may wash away into the coarse material. Over a period of time, this process may clog the void spaces in the coarser material. Such a situation can be prevented by the use of a filter or protective filter between the two soils. For example, consider the earth dam section shown in Figure 2.66. If rock fills were only used at the toe of the dam, the seepage water would wash the fine soil grains into the toe and undermine the structure. Hence, for the safety of the structure, a filter should be placed between the fine soil and the rock toe (Figure 2.69). For the proper selection of the filter material, two conditions should be kept in mind:

![Figure 2.68 Flow net for seepage through a zoned earth dam](image)
1. The size of the voids in the filter material should be small enough to hold the larger particles of the protected material in place.
2. The filter material should have a high permeability to prevent building of large seepage forces and hydrostatic pressure in the filter.

Based on the experimental investigation of protective filters, Bertram (1940) provided the following criteria to satisfy the above condition:

\[
\frac{D_{15(F)}}{D_{85(S)}} \leq 4 \text{ to } 5 \quad \text{(to satisfy condition 1)} \tag{2.200}
\]

\[
\frac{D_{15(F)}}{D_{15(S)}} \leq 4 \text{ to } 5 \quad \text{(to satisfy condition 2)} \tag{2.201}
\]

Where

\[D_{15(F)} = \text{diameter through which 15% of filter material will pass}\]

\[D_{15(S)} = \text{diameter through which 15% of soil to be protected will pass}\]

\[D_{85(S)} = \text{diameter through which 85% of soil to be protected will pass}\]

The proper use of equation (2.200) and (2.201) to determine the grain-size distribution of soils used as filters is shown in Figure 2.70. Consider the soil used for the construction of the earth dam shown in Figure 2.69. Let the grain-size distribution of this soil be given by curve a in Figure 2.70. We can
now determine $5D_{85(S)}$ and $5D_{15(S)}$ and plot them as shown in Figure 2.70. The acceptable grain-size distribution of the filter material will have to lie in the shaded zone.

![Figure 2.70 Determination of grain-size distribution of filter using eqs. (2.200) and (2.201)](image)

The same principle can be adopted for determination of the size limits for the rock layer (Figure 2.69) to protect the filter material from being washed away.

The U. S. Navy (1971) requires the following conditions for the design of filters.

1. For avoiding the movement of the particles of the protected soil:

$$\frac{D_{15(F)}}{D_{85(S)}} < 5$$

$$\frac{D_{50(F)}}{D_{50(S)}} < 25$$

$$\frac{D_{15(F)}}{D_{15(S)}} < 20$$

If the uniformity coefficient $C_u$ of the protected soil is less than 1.5, $D_{15(F)}/D_{85(S)}$ may be increased to 6. Also, if $C_u$ of the protected soil is greater than 4, $D_{15(F)}/D_{15(S)}$ may be increased to 40.

2. For avoiding buildup of large seepage force in the filter,

$$\frac{D_{15(F)}}{D_{15(S)}} > 4$$

3. The filter material should not have grain sizes greater than 3 in (76.2 mm). (This is to avoid segregation of particles in the filter.)
4. To avoid internal movement of fines in the filter, it should have no more than 5% passing a No. 200 sieve.

5. When perforated pipes are used for collecting seepage water, filters are also used around the pipes to protect the fine-grained soil from being washed into the pipes. To avoid the movement of the filter material into the drain-pipe perforations, the following additional conditions should be met:

\[
\frac{D_{15(F)}}{\text{slot width}} > 1.2 \text{ to } 1.4
\]

\[
\frac{D_{85(F)}}{\text{hole diameter}} > 1.0 \text{ to } 1.2
\]

Thanikachalam and Sakthivadivel (1974) analyzed experimental results for filters reported by Karpoff (1955), U. S. Corps of Engineers (1953), Leartherwood and Peterson (1954), Dayaprakash and Gupta (1972), and Belyashevskii et al. (1972). Based on this analysis, they recommended that when the soil to be protected is of a granular nature, the stable filter design criteria may be given by the following equations:

\[
\frac{D_{60(S)}}{D_{10(S)}} = 0.4 \left( \frac{D_{10(F)}}{D_{10(S)}} - 2.0 \right)
\]  
(2.202)

And

\[
\frac{D_{60(F)}}{D_{10(F)}} = 0.941 \left( \frac{D_{10(F)}}{D_{10(S)}} - 5.65 \right)
\]  
(2.203)

Where \(D_{60(S)}\) and \(D_{10(S)}\) are, respectively, the diameters through which 60% and 10% of the soil to be protected in passing; and \(D_{60(F)}\) and \(D_{10(F)}\) are, respectively, the diameters through which 60% and 10% of the filter material is passing.

Cedegren (1960) constructed several flow nets, such as those shown in Figure 2.71a, and b, to study the condition of seepage into sloping filters placed at the downstream side of earth dams. Based on this work, he developed the chart given in Figure 2.71c which allows us to determine the minimum thickness of filter material, \(W\), required on the downstream side of an earth dam. (Note that in Figure 2.71, \(k_F\) is the coefficient of permeability of the filter material, and \(k_S\) is the coefficient of permeability of the soil of the earth dam.)
Figure 2.71 Thickness of filter material on the downstream side of an earth dam. [After H. R. Cedergren, Seepage Requirement of Filters and Pervious Bases, J. Soil Mech. Found. Div., ASCE, vol. 86, no. SM5 (part I), 1960.]