Lesson 9

Doubly Reinforced Beams – Theory
Instructional Objectives:

At the end of this lesson, the student should be able to:

- design the amounts of compression and tensile reinforcement if the \( b, d, d', f_{ck}, f_y \) and \( M_u \) are given, and
- determine the moment of resistance of a beam if \( b, d, d', f_{ck}, f_y, A_{sc} \) and \( A_{st} \) are given.

4.9.1 Introduction

This lesson illustrates the application of the theory of doubly reinforced beams in solving the two types of problems mentioned in Lesson 8. Both the design and analysis types of problems are solved by (i) direct computation method, and (ii) using tables of SP-16. The step by step solution of the problems will help in understanding the theory of Lesson 8 and its application.

4.9.2 Numerical problems

4.9.2.1 Problem 4.1

Design a simply supported beam of effective span 8 m subjected to imposed loads of 35 kN/m. The beam dimensions and other data are: \( b = 300 \text{ mm}, \ D = 700 \text{ mm}, \ M 20 \text{ concrete}, \ Fe 415 \text{ steel (Fig. 4.9.1)}. \) Determine \( f_{sc} \) from \( d'/d \) as given in Table 4.2 of Lesson 8.
(a) Solution by direct computation method

Dead load of the beam = 0.3 (0.7) (25) = 5.25 kN/m

Imposed loads (given) = 35.00 kN/m

Total loads = 5.25 + 35.00 = 40.25 kN/m

Factored bending moment = \( \frac{1.5 \times wL^2}{8} = \frac{(1.5)(40.25)(8)}{8} = 482.96 \) kNm

Assuming \( d' = 70 \) mm, \( d = 700 - 70 = 630 \) mm

\[ \frac{x_{u, \text{max}}}{d} = 0.48 \text{ gives } x_{u, \text{max}} = 0.48 (630) = 302.4 \text{ mm} \]

Step 1: Determination of \( M_{u, \lim} \) and \( A_{st, \lim} \)

\[ M_{u, \lim} = 0.36 \left( \frac{x_{u, \text{max}}}{d} \right) (1 - 0.42 \frac{x_{u, \text{max}}}{d}) b d^2 f_{ck} \]

(4.2)

\[ = 0.36(0.48) \{1 - 0.42 (0.48)\} (300) (630)^2 (20) (10^{-6}) \text{ kNm} \]

\[ = 328.55 \text{ kNm} \]

\[ A_{st, \lim} = \frac{M_{u, \lim}}{0.87 f_y (d - 0.42 x_{u, \text{max}})} \]

(6.8)

So,

\[ A_{st1} = \frac{328.55 (10^6) \text{ Nmm}}{0.87 (415) \{630 - 0.42 (0.48) 630\}} = 1809.14 \text{ mm}^2 \]

Step 2: Determination of \( M_{u2}, A_{scr}, A_{st2} \) and \( A_{st} \)

(Please refer to Eqs. 4.1, 4.4, 4.6 and 4.7 of Lesson 8.)

\[ M_{u2} = M_u - M_{u, \lim} = 482.96 - 328.55 = 154.41 \text{ kNm} \]

Here, \( d'/d = 70/630 = 0.11 \)

From Table 4.2 of Lesson 8, by linear interpolation, we get,
\[ f_{sc} = 353 - \frac{353 - 342}{5} = 350.8 \text{ N/mm}^2 \]

\[ A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')} = \frac{154.41 \times 10^6 \text{ Nmm}}{(350.8 - 0.446)(20)(630 - 70) \text{ N/mm}} = 806.517 \text{ mm}^2 \]

\[ A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{806.517 (350.8 - 8.92)}{(0.87) (415)} = 763.694 \text{ mm}^2 \]

\[ A_{st} = A_{st1} + A_{st2} = 1809.14 + 783.621 = 2572.834 \text{ mm}^2 \]

**Step 3:** Check for minimum and maximum tension and compression steel.

 voi sec.4.8.5 of Lesson 8

(i) In compression:

(a) Minimum \( A_{sc} = \frac{0.2}{100} (300)(700) = 420 \text{ mm}^2 \)

(b) Maximum \( A_{sc} = \frac{4}{100} (300)(700) = 8400 \text{ mm}^2 \)

Thus, 420 mm\(^2\) < 806.517 mm\(^2\) < 8400 mm\(^2\) . Hence, o.k.

(ii) In tension:

(a) Minimum \( A_{st} = \frac{0.85 b d}{f_y} \times \frac{0.85(300)(630)}{415} = 387.1 \text{ mm}^2 \)

(b) Maximum \( A_{st} = \frac{4}{100} (300)(700) = 8400 \text{ mm}^2 \)

Here, 387.1 mm\(^2\) < 2572.834 mm\(^2\) < 8400 mm\(^2\) . Hence, o.k.

**Step 4:** Selection of bar diameter and numbers.

(i) for \( A_{sc} \): Provide 2-20 T + 2-12 T (= 628 + 226 = 854 mm\(^2\) )

(ii) for \( A_{st} \): Provide 4-25 T + 2-20 T (= 1963 + 628 = 2591 mm\(^2\) )

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It may be noted that $A_{st}$ is provided in two layers in order to provide adequate space for concreting around reinforcement. Also the centroid of the tensile bars is at 70 mm from bottom (Fig. 4.9.1).

(b) Solution by use of table of SP-16

For this problem, \[ \frac{M_u}{b d^2} = \frac{482.96 (10^6)}{300 (630)^2} = 4.056 \] and \[ d'/d = \frac{70}{630} = 0.11. \]

Table 50 of SP-16 gives $p_t$ and $p_c$ for \[ \frac{M_u}{b d^2} = 4 \text{ and } 4.1 \text{ and } d'/d = 0.1 \text{ and } 0.15. \]

The required $p_t$ and $p_c$ are determined by linear interpolation. The values are presented in Table 4.3 to get the final $p_t$ and $p_c$ of this problem.

Table 4.3 Calculation of $p_t$ and $p_c$

<table>
<thead>
<tr>
<th>$\frac{M_u}{b d^2}$</th>
<th>$d'/d = 0.1$</th>
<th>$d'/d = 0.15$</th>
<th>$d'/d = 0.11$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>1.337</td>
<td>1.360</td>
<td>1.337 + 0.023 (0.01) / 0.05 = 1.342</td>
</tr>
<tr>
<td></td>
<td>0.401</td>
<td>0.437</td>
<td>0.433 + 0.036 (0.01) / 0.05 = 0.408</td>
</tr>
<tr>
<td>4.1</td>
<td>1.368</td>
<td>1.392</td>
<td>1.368 + 0.024 (0.01) / 0.05 = 1.373</td>
</tr>
<tr>
<td></td>
<td>0.433</td>
<td>0.472</td>
<td>0.433 + 0.039 (0.01) / 0.05 = 0.441</td>
</tr>
<tr>
<td>4.056</td>
<td>Not Applicable (NA)</td>
<td>NA</td>
<td>1.342 + 0.031 (0.056) / 0.1 = 1.3594</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>NA</td>
<td>0.408 + 0.033 (0.056) / 0.1 = 0.426</td>
</tr>
</tbody>
</table>

So, \[ A_{st} = \frac{1.3594 (300) (630)}{100} = 2569.26 \text{ m}^2 \]

and \[ A_{sc} = \frac{0.426 (300) (630)}{100} = 805.14 \text{ mm}^2 \]

These values are close to those obtained by direct computation method where \[ A_{st} = 2572.834 \text{ mm}^2 \text{ and } A_{sc} = 806.517 \text{ mm}^2. \] Thus, by using table of SP-16 we
get the reinforcement very close to that of direct computation method. Hence, provide

(i) for $A_{sc}$: $20 T + 212 T (= 628 + 226 = 854 \text{ mm}^2)$

(ii) for $A_{st}$: $25 T + 220 T (= 1963 + 628 = 2591 \text{ mm}^2)$

4.9.2.2 Problem 4.2

Determine the ultimate moment capacity of the doubly reinforced beam of $b = 350 \text{ mm}$, $d' = 60 \text{ mm}$, $d = 600 \text{ mm}$, $A_{st} = 2945 \text{ mm}^2$ (6-25 T), $A_{sc} = 1256 \text{ mm}^2$ (4-20 T), using M 20 and Fe 415 (Fig.4.9.2). Use direct computation method only.

Solution by direct computation method

Step 1: To check if the beam is under-reinforced or over-reinforced.

$$x_{n, \text{max}} = 0.48 (600) = 288 \text{ mm}$$

$$\varepsilon_{st} = \frac{\varepsilon_c (d - x_{n, \text{max}})}{x_{sc, \text{max}}} = \frac{0.0035 (600 - 288)}{288} = 0.00379$$
Yield strain of Fe 415 = \( \frac{f_y}{1.15(E_s)} + 0.002 = \frac{415}{(1.15)(2)(10^5)} + 0.002 \)

\[ = 0.0038 > 0.00379. \]

Hence, the beam is over-reinforced.

**Step 2: To determine** \( M_{u,lim} \) and \( A_{st,lim} \)
(vide Eq. 4.2 of Lesson 8 and Table 3.1 of Lesson 5)

\[
M_{u,lim} = 0.36 \left( \frac{x_{u, \text{max}}}{d} \right) (1 - 0.42 \frac{x_{u, \text{max}}}{d}) b d^2 f_{ck} 
\]

\[
= 0.36 (0.48) \{1 - 0.42 (0.48)\} (350) (600)^2 (20) (10^6) \text{ kNm} 
\]

\[ = 347.67 \text{ kNm} \]

From Table 3.1 of Lesson 5, for \( f_{ck} = 20 \text{ N/mm}^2 \) and \( f_y = 415 \text{ N/mm}^2 \),

\[
A_{st,lim} = \frac{0.96 (350)(600)}{100} = 2016 \text{ mm}^2
\]

**Step 3: To determine** \( A_{st2} \) and \( A_{sc} \)
(vide Eqs.4.7 and 4.6 of Lesson 8)

\[
A_{st2} = A_{st} - A_{st,lim} = 2945 - 2016 = 929 \text{ mm}^2
\]

The required \( A_{sc} \) will have the compression force equal to the tensile force as given by 929 mm\(^2\) of \( A_{st2} \).

So, \[
A_{sc} = \frac{A_{st2} (0.87 f_y)}{(f_{sc} - f_{cc})}
\]

For \( f_{sc} \) let us calculate \( \varepsilon_{sc} \): (vide Eq. 4.9 of Lesson 8)

\[
\varepsilon_{sc} = \frac{0.0035 (x_{u, \text{max}} - d')}{x_{u, \text{max}}} = \frac{0.0035 (288 - 60)}{288} = 0.002771
\]

Table 4.1 of Lesson 8 gives:

\[
f_{sc} = 351.8 + \frac{(360.9 - 351.8)(0.002771 - 0.002760)}{(0.00380 - 0.00276)} = 351.896 \text{ N/mm}^2
\]
So, \[ A_{sc} = \frac{929 (0.87) (415)}{351.89 - 0.446 (20)} = 977.956 \ mm^2 \]

Step 4: To determine \( M_{u2} \), \( M_u \) and \( A_{st} \)
(Please refer to Eqs. 4.4 and 4.1 of Lesson 8)

\[ M_{u2} = A_{sc} (f'_{sc} - f_{sc}) (d - d') \]
\[ = 977.956 \{351.896 - 0.446 (20)\} (600 - 60) (10^{-6}) \ kNm \]
\[ = 181.12 \ kNm \]

\[ M_u = M_{u,lim} + M_{u2} = 347.67 + 181.12 = 528.79 \ kNm \]

Therefore, with \( A_{st} = A_{st,lim} + A_{st2} = 2016 + 929 = 2945 \ mm^2 \) the required \( A_{sc} = 977.956 \ mm^2 \) (much less than the provided 1256 mm$^2$). Hence, o.k.

4.9.3 Practice Questions and Problems with Answers

Q.1: Design a doubly reinforced beam (Fig. 4.9.3) to resist \( M_u = 375 \ kNm \) when \( b = 250 \ mm, \ d = 500 \ mm, \ d' = 75 \ mm, \ f_{ck} = 30 \ N/mm^2 \) and \( f_y = 500 \ N/mm^2 \), using (i) direct computation method and (ii) using table of SP-16.
A.1: (A) Solution by direct computation method:

From the given data

\[
M_{u, \text{lim}} = 0.36 \left( \frac{x_{u, \text{max}}}{d} \right) \left( 1 - 0.42 \frac{x_{u, \text{max}}}{d} \right) b d^2 f_{ck}
\]

\[
= 0.36 (0.46) \{1 - 0.42 (0.46)\} (250) (500)^2 (30) (10^{-6}) \text{kNm}
\]

\[
= 250.51 \text{kNm}
\]

Using the value of \( p_t = 1.13 \) from Table 3.1 of Lesson 5 for \( f_{ck} = 30 \text{N/mm}^2 \) and \( f_y = 500 \text{N/mm}^2 \),

\[
A_{st, \text{lim}} = \frac{1.13 (250)(500)}{100} = 1412.5 \text{mm}^2
\]

\[
M_{u2} = 375 - 250.51 = 124.49 \text{kNm}
\]

From Table 4.2 of Lesson 8, for \( d'/d = 75/500 = 0.15 \) and \( f_y = 500 \text{N/mm}^2 \), we get \( f_{sc} = 395 \text{N/mm}^2 \)

\[
A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc}) (d - d')} = \frac{124.49 (10^6)}{\{395 - 0.446 (30)\} (500 - 75)} = 767.56 \text{mm}^2
\]

\[
A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} = \frac{767.56 \{395 - 0.446 (30)\}}{0.87 (500)} = 673.37 \text{mm}^2
\]

\[
A_{st} = A_{st, \text{lim}} + A_{st2} = 1412.5 + 673.37 = 2085.87 \text{mm}^2
\]

Alternatively: (use of Table 4.1 of Lesson 8 to determine \( f_{sc} \) from \( \varepsilon_{sc} \))

\[
x_{u, \text{max}} = 0.46 (500) = 230 \text{ mm}
\]

\[
\varepsilon_{sc} = \frac{0.0035 (230 - 75)}{230} = \frac{0.0035 (155)}{230} = 0.002359
\]

From Table 4.1
\[ f_{sc} = 391.3 + \frac{(413.0 - 391.3)(0.002359 - 0.00226)}{(0.00277 - 0.00226)} = 395.512 \text{ N/mm}^2 \]

\[ A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')} = \frac{124.49 \times 10^6}{(395.512 - 0.446)(30)(500 - 75)} = 766.53 \text{ mm}^2 \]

\[ A_{st,2} = \frac{A_e f_{sc} - f_{cc}}{0.87f_y} = \frac{766.53(382.132)}{0.87(500)} = 673.369 \text{ mm}^2 \]

\[ A_{st} = A_{st,lim} + A_{st2} = 1412.5 + 673.369 = 2085.869 \text{ mm}^2 \]

Check for minimum and maximum \(A_{st}\) and \(A_{sc}\)

(i) Minimum \(A_{st} = \frac{0.85bD}{f_y} = \frac{0.85(250)(500)}{500} = 212.5 \text{ mm}^2\)

(ii) Maximum \(A_{st} = 0.04bD = 0.04(250)(575) = 5750 \text{ mm}^2\)

(iii) Minimum \(A_{st} = \frac{0.2bD}{100} = \frac{0.2(250)(575)}{100} = 287.5 \text{ mm}^2\)

(iv) Maximum \(A_{st} = 0.04bD = 0.04(250)(575) = 5750 \text{ mm}^2\)

Hence, the areas of reinforcement satisfy the requirements.

So, provide (i) 6-20 T + 2-12 T = 1885 + 226 = 2111 mm² for \(A_{st}\)

(ii) 4-16 T = 804 mm² for \(A_{sc}\)

(B) Solution by use of table of SP-16

From the given data, we have

\[ \frac{M_u}{b.d^2} = \frac{375 \times 10^6}{250(500)^2} = 6.0 \]
\[ d'/d = \frac{75}{500} = 0.15 \]

Table 56 of SP-16 gives: \( p_t = 1.676 \) and \( p_c = 0.619 \)

So,
\[
A_{st} = \frac{(1.676)(250)(500)}{100} = 2095 \text{ mm}^2
\]

and
\[
A_{sc} = \frac{(0.619)(250)(500)}{100} = 773.75 \text{ mm}^2
\]

These values are close to those of (A). Hence, provide 6-20 T + 2-12 T as \( A_{st} \) and 4-16 T as \( A_{sc} \).

Q.2: Determine the moment of resistance of the doubly reinforced beam (Fig. 4.9.4) with \( b = 300 \text{ mm}, \ d = 600 \text{ mm}, \ d' = 90 \text{ mm}, \ f_{ck} = 30 \text{ N/mm}^2, \ f_y = 500 \text{ N/mm}^2, \ A_{sc} = 2236 \text{ mm}^2 \ (2-32 \text{ T} + 2-20 \text{ T}), \) and \( A_{st} = 4021 \text{ mm}^2 \ (4-32 \text{ T} + 4-16 \text{ T}) \). Use (i) direct computation method and (ii) tables of SP-16.

A.2: (i) Solution by direct computation method:

\[
x_{u,\text{max}} = 0.46 (600) = 276 \text{ mm}
\]
\[ \varepsilon_{st} = \frac{0.0035 \times (600 - 276)}{276} = 0.0041086 \]

\[ \varepsilon_{yield} = 0.00417. \] So \( \varepsilon_{st} < \varepsilon_{yield} \) i.e. the beam is over-reinforced.

For \( d''/d = 0.15 \) and \( f_y = 500 \text{ N/mm}^2 \), Table 4.2 of Lesson 8 gives: \( f_{sc} = 395 \text{ N/mm}^2 \) and \( f_{ck} = 30 \text{ N/mm}^2 \), Table 3.1 of Lesson 5 gives \( p_{t,\text{lim}} = 1.13 \).

\[ A_{st,\text{lim}} = \frac{1.13 \times 300 \times 600}{100} = 2034 \text{ mm}^2 \]

\[ M_{u,\text{lim}} = 0.36 \left( \frac{x_{u,\text{max}}}{d} \right) (1 - 0.42 \frac{x_{u,\text{max}}}{d}) b d^2 f_{ck} \]

\[ = 0.36 (0.46) \{1 - 0.42 (0.46)\} (300) (600)^2 (30) (10^{-6}) \text{ kNm} \]

\[ = 432.88 \text{ kNm} \]

\[ A_{st2} = 4021 - 2034 = 1987 \text{ mm}^2 \]

\[ (A_{sc})_{\text{required}} = \frac{A_{st2} (0.87) f_y}{(f_{sc} - f_{cc})} = \frac{1987 (0.87) (500)}{\{395 - 0.446 (30)\}} = 2264.94 \text{ mm}^2 > 2236 \text{ mm}^2 \]

So, \( A_{st2} \) of 1987 mm\(^2\) is not fully used. Let us determine \( A_{st2} \) required when \( A_{sc} = 2236 \text{ mm}^2 \).

\[ A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} = \frac{2236 \{395 - 0.446 (30)\}}{(0.87) (500)} = 1961.61 \text{ mm}^2 \]

\[ A_{st} = A_{st,\text{lim}} + A_{st2} = 2034 + 1961.61 = 3995.61 \text{ mm}^2 < 4021 \text{ mm}^2. \]

Hence, o.k.

With \( A_{st2} = 1961.61 \text{ mm}^2 \), \( M_{u2} = A_{st2} (0.87 \ f_y \ ) (d - d' ) \)

\[ = 1961.61 (0.87) (500) (600 - 75) (10^{-6}) \text{ kNm} = 447.98268 \text{ kNm} \]

Again, when \( A_{sc} = 2236 \text{ mm}^2 \) (as provided)
\[ M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d - d') \]
\[ = 2236 \{395 - 0.446 (30)\} (600 - 75) (10^{-6}) \text{ kNm} = 447.9837 \text{ kNm} \]

\[ M_u = M_{u, \text{lim}} + M_{u2} = 432.88 + 447.98 \text{ (} M_{u2} \text{ is taken the lower of the two)} \]
\[ = 880.86 \text{ kNm} \]

Hence, the moment of resistance of the beam is 880.86 kNm.

Alternatively \( f_{sc} \) can be determined from Table 4.1 of Lesson 8.

Using the following from the above:

\[ x_{u, \text{max}} = 276 \text{ mm} \]
\[ A_{st, \text{lim}} = 2034 \text{ mm}^2 \]
\[ M_{u, \text{lim}} = 432.88 \text{ kNm} \]
\[ A_{st2} = 1987 \text{ mm}^2 \]

To find \( (A_{sc})_{\text{required}} \)

\[ \varepsilon_{st} = \frac{0.0035 (276 - 90)}{276} = 0.00236 \]

Table 4.1 of Lesson 8 gives:

\[ f_{sc} = 391.3 + \frac{(413 - 391.3) (0.00236 - 0.00226)}{(0.00277 - 0.00226)} = 395.55 \text{ N/mm}^2 \]

\[ (A_{sc})_{\text{required}} = \frac{A_{st2} (0.87) f_y}{(f_{sc} - f_{cc})} = \frac{1987 (0.87) (500)}{\{395.55 - 0.446 (30)\}} \]
\[ = 2261.68 \text{ mm}^2 > 2236 \text{ mm}^2 \]

So, it is not o.k.

Let us determine \( A_{st2} \) required when \( A_{sc} = 2236 \text{ mm}^2 \).
\[ A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} = \frac{2236 \{395.55 - 0.446 (30)\}}{(0.87) (500)} = 1964.44 \text{ mm}^2 \]

\[ A_{st} = A_{st, \text{lim}} + A_{st2} = 2034 + 1964.44 = 3998.44 \text{ mm}^2 < 4021 \text{ mm}^2. \]

So, o.k.

\[ M_{u2} \text{ (when } A_{st2} = 1964.44 \text{ mm}^2) = A_{st2} (0.87 f_y) (d - d') \]

\[ = 1964.44 (0.87) (500) (600 - 75) (10^{-6}) \text{ kNm} \]

\[ = 448.63 \text{ kNm} \]

For \( A_{sc} = 2236 \text{ mm}^2, \)

\[ M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d - d') \]

\[ = 2236 \{395.55 - 0.446 (30)\} (600 - 75) (10^{-6}) \text{ kNm} \]

\[ = 2236 (382.17) (525) (10^{-6}) \text{ kNm} \]

\[ = 448.63 \text{ kNm} \]

Both the \( M_{u2} \) values are the same. So,

\[ M_u = M_{u, \text{lim}} + M_{u2} = 432.88 + 448.63 \]

\[ = 881.51 \text{ kNm} \]

Here, the \( M_u = 881.51 \text{ kNm}. \)

(ii) Solution by using table of SP-16

From the given data:

\[ p_i = \frac{4021(100)}{300(600)} = 2.234 \]

\[ p_c = \frac{2236(100)}{300(600)} = 1.242 \]

\[ d'/d = 0.15 \]
Table 56 of SP-16 is used first considering \( d'/d = 0.15 \) and \( p_t = 2.234 \), and secondly, considering \( d'/d = 0.15 \) and \( p_c = 1.242 \). The calculated values of \( p_c \) and \( M_u/bd^2 \) for the first and \( p_t \) and \( M_u/bd^2 \) for the second cases are presented below separately. Linear interpolation has been done.

(i) When \( d'/d = 0.15 \) and \( p_t = 2.234 \)

\[
\frac{M_u}{b d^2} = 8.00 + \frac{(8.1-8.0) (2.234 - 2.218)}{(2.245 - 2.218)} = 8.06
\]

\[
p_c = 1.235 + \frac{(1.266-1.235)(0.016)}{(0.027)} = 1.253 > 1.242
\]

So, this is not possible.

(ii) When \( d'/d = 0.15 \) and \( p_c = 1.242 \)

\[
\frac{M_u}{b d^2} = 8.00 + \frac{(8.1-8.0) (1.242 -1.235)}{(1.266 -1.235)} = 8.022
\]

\[
p_t = 2.218 + \frac{(2.245 - 2.218) (1.242 -1.235)}{(1.266 - 1.235)} = 2.224 < 2.234
\]

So, \( M_u = 8.022 (300) (600)^2 (10^{-6}) = 866.376 \) kNm.

Hence, o.k.

4.9.4 References


4.9.5 Test 9 with Solutions

Maximum Marks = 50,  Maximum Time = 30 minutes

Answer all questions.

TQ.1: Design a simply supported beam of effective span 8 m subjected to imposed loads of 35 kN/m. The beam dimensions and other data are: \( b = 300 \text{ mm} \), \( D = 700 \text{ mm} \), M 20 concrete, Fe 415 steel (Fig. 4.9.1). Determine \( f_{sc} \) from strain \( \varepsilon_{sc} \) as given in Table 4.1 of Lesson 8.

A.TQ.1: This problem is the same as Problem 4.1 in sec. 4.9.2.1 except that here the \( f_{sc} \) is to be calculated using Table 4.1 instead of Table 4.2.

Step 1: Here, the Step 1 will remain the same as that of Problem 4.1.

Step 2: Determination of \( M_{u2} \), \( A_{sc} \), \( A_{st2} \) and \( A_{st} \)

\[
M_{u2} = M_u - M_{u, \text{lim}} = 482.96 - 328.55 = 154.41 \text{ kNm}
\]
From strain triangle: (Fig. 4.8.2 of Lesson 8)

\[ \varepsilon_{sc} = \frac{0.0035 (302.4 - 70)}{302.4} = 0.00269 \]

\[ f_{sc} \text{ (from Table 4.1 of Lesson 8)} = 342.8 + \frac{(351.8 - 342.8)}{(0.00276 - 0.00241)} (0.00269 - 0.00241) \]

\[ = 350 \text{ N/mm}^2 \]

\[ A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc}) (d - d') = \frac{154.41(10^6)}{(350 - 0.446(20)) (630 - 70) \text{ N/mm}}} = 808.41 \text{ mm}^2 \]

\[ A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} = \frac{808.41 (341.08)}{(0.87) (415)} = 763.696 \text{ mm}^2 \]

\[ A_{st} = A_{st1} + A_{st2} = 1809.14 + 763.696 = 2572.836 \text{ mm}^2 \]

\[ A_{sc} = 808.41 \text{ mm}^2 \]

Steps 3 & 4 will also remain the same as those of Problem 4.1.

Hence, provide 2-20 T + 2-12 T (854 mm\(^2\)) as \(A_{sc}\) and 4-25 T + 2-20 T (2591 mm\(^2\)) as \(A_{st}\).

TQ.2: Determine the ultimate moment capacity of the doubly reinforced beam of 

\[ b = 350 \text{ mm}, \quad d' = 60 \text{ mm}, \quad d = 600 \text{ mm}, \quad A_{st} = 2945 \text{ mm}^2 (6-25 T), \quad A_{sc} = 1256 \text{ mm}^2 (4-20 T), \]  

using M 20 and Fe 415 (Fig. 4.9.2). Use table of SP-16 only.

A.TQ.2: Solution by using table of SP-16

This problem is the same as that of Problem 4.2 of sec. 4.9.2.2, which has been solved by direct computation method. Here, the same is to be solved by using SP-16.

The needed parameters are:

\[ d'/d = 60/600 = 0.1 \]

\[ p = \frac{A_{st} (100)}{b d} = \frac{2945 (100)}{350(600)} = 1.402 \]
\[ p_c = \frac{A_{sc}(100)}{b \cdot d} = \frac{1256(100)}{350(600)} = 0.5981 \]

Here, we need to use Table 50 for \( f_{ck} = 20 \, \text{N/mm}^2 \) and \( f_y = 415 \, \text{N/mm}^2 \). The table gives values of \( M_u/\beta d^2 \) for (i) \( d'/d \) and \( p_t \) and (ii) \( d'/d \) and \( p_c \). So, we will consider both the possibilities and determine \( M_u \).

(i) Considering Table 50 of SP-16 when \( d'/d = 0.1 \) and \( p_t = 1.402 \):

Interpolating the values of \( M_u/\beta d^2 \) at \( p_t = 1.399 \) and 1.429, we get

\[
\left( \frac{M_u}{b \cdot d^2} \right)_{p_t=1.402} = 4.2 + \frac{(4.3-4.2)(1.402-1.399)}{(1.429-1.399)} = 4.21
\]

the corresponding \( (p_c)_{p_t=1.402} \) is:

\[
0.466 + \frac{(0.498-0.466)(1.402-1.399)}{(1.429-1.399)} = 0.4692
\]

But, \( p_c \) provided is 0.5981 indicates that extra compression reinforcement has been used.

So, we get

\[ M_u = 4.21 \cdot b \cdot d^2 = (4.21)(350)(600)^2 (10^{-6}) = 530.46 \, \text{kNm} \text{ when } A_{st} = 2945 \, \text{mm}^2 \text{ and } A_{sc} = 985.32 \, \text{mm}^2, \text{ i.e. } 270.69 \, \text{mm}^2 (= 1256 - 985.32) \text{ of compression steel is extra.} \]

(ii) Considering \( d'/d = 0.1 \) and \( p_c = 0.5981 \), we get by linear interpolation

\[
\left( \frac{M_u}{b \cdot d^2} \right)_{p_c=0.5981} = 4.6 + \frac{(4.7-4.6)(0.5981-0.595)}{(0.628-0.595)} = 4.61
\]

the corresponding \( p_t \) is:

\[
(p_t)_{p_c=0.5981} = 1.522 + \frac{(1.533-1.522)(0.5981-0.595)}{(0.628-0.595)} = 1.5231
\]

The provided \( p_t = 1.402 \) indicates that the tension steel is insufficient by 254.31 mm\(^2\) as shown below:

\[ \text{Amount of additional } A_{st} \text{ still required } = \]
If this additional steel is provided, then the \( M_u \) of this beam becomes:

\[
M_u = 4.61 \cdot b \cdot d^2 = 4.61 \cdot (350) \cdot (600)^2 \cdot (10^{-6}) \text{ kNm} = 580.86 \text{ kNm}
\]

The above two results show that the moment of resistance of this beam is the lower of the two. So, \( M_u = 530.46 \text{ kNm} \). By direct computation the \( M_u = 528.79 \text{ kNm} \). The two results are in good agreement.

### 4.9.6 Summary of this Lesson

This lesson presents solutions of four numerical problems covering both design and analysis types. These problems are solved by two methods: (i) direct computation method and (ii) using table of SP-16. Two problems are illustrated in the lesson and the other two are given in the practice problem and test of this lesson. The solutions will help in understanding the step by step application of the theory of doubly reinforced beams given in Lesson 8.