Module 5

Flanged Beams – Theory and Numerical Problems
Lesson 12
Flanged Beams – Numerical Problems (Continued)
Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify the two types of problems – analysis and design types,
- apply the formulations to design the flanged beams.

5.12.1 Introduction

Lesson 10 illustrates the governing equations of flanged beams and Lesson 11 explains their applications for the solution of analysis type of numerical problems. It is now necessary to apply them for the solution of design type, the second type of the numerical problems. This lesson mentions the different steps of the solution and solves several numerical examples to explain their step-by-step solutions.

5.12.2 Design Type of Problems

We need to assume some preliminary dimensions of width and depth of flanged beams, spacing of the beams and span for performing the structural analysis before the design. Thus, the assumed data known for the design are: $D_f$, $b_w$, $D$, effective span, effective depth, grades of concrete and steel and imposed loads.

There are four equations: (i) expressions of compressive force $C$, (ii) expression of the tension force $T$, (iii) $C = T$ and (iv) expression of $M_u$ in terms of $C$ or $T$ and the lever arm ($M = (C$ or $T)$ (lever arm)). However, the relative dimensions of $D_f$, $D$ and $x_u$ and the amount of steel (under-reinforced, balanced or over-reinforced) influence the expressions. Accordingly, the respective equations are to be employed assuming a particular situation and, if necessary, they need to be changed if the assumed parameters are found to be not satisfactory. The steps of the design problems are as given below.

**Step 1:** To determine the factored bending moment $M_u$

**Step 2:** To determine the $M_{u,lim}$ of the given or the assumed section

The beam shall be designed as under-reinforced, balanced or doubly reinforced if the value of $M_u$ is less than, equal to or more than $M_{u,lim}$. The design of over-reinforced beam is to be avoided as it does not increase the bending moment carrying capacity beyond $M_{u,lim}$ either by increasing the depth or designing a doubly reinforced beam.

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Step 3: To determine $x_u$, the distance of the neutral axis, from the expression of $M_u$

Here, it is necessary to assume first that $x_u$ is in the flange. Later on, it may be necessary to calculate $x_u$ if the value is found to be more than $D_f$. This is to be done assuming first that $D_f/x_u < 0.43$ and then $D_f/x_u > 0.43$ separately.

Step 4: To determine the area(s) of steel

For doubly reinforced beams $A_{st} = A_{st,lim} + A_{st2}$ and $A_{sc}$ are to be obtained, while only $A_{st}$ is required to be computed for under-reinforced and balanced beams. These are calculated employing $C = T$ (for $A_{st}$ and $A_{st,lim}$) and the expression of $M_{u2}$ to calculate $A_{st2}$ and $A_{sc}$.

Step 5: It may be necessary to check the $x_u$ and $A_{st}$ once again after Step 4

It is difficult to prescribe all the relevant steps of design problems. Decisions are to be taken judiciously depending on the type of problem. For the design of a balanced beam, it is necessary to determine the effective depth in Step 3 employing the expression of bending moment $M_u$. For such beams and for under-reinforced beams, it may be necessary to estimate the $A_{st}$ approximately immediately after Step 2. This value of $A_{st}$ will facilitate to determine $x_u$.

5.12.3 Numerical Problems

Four numerical examples are solved below explaining the steps involved in the design problems.
Ex.5: Design the simply supported flanged beam of Fig. 5.12.1, given the following: $D_f = 100$ mm, $D = 750$ mm, $b_w = 350$ mm, spacing of beams = 4000 mm c/c, effective span = 12 m, cover = 90 mm, $d = 660$ mm and imposed loads = 5 kN/m². Fe 415 and M 20 are used.

Solution:

**Step 1: Computation of factored bending moment**

Weight of slab per m$^2 = (0.1)(1)(1)(25) = 2.5$ kN/m$^2$

So, Weight of slab per m = (4)(2.5) = 10.00 kN/m

Dead loads of web part of the beam = (0.35)(0.65)(1)(25) = 5.6875 kN/m

Imposed loads = (4)(5) = 20 kN/m

Total loads = 30 + 5.6875 = 35.6875 kN/m

Factored Bending moment = $(1.5) \frac{(35.6875)(12)(12)}{8} = 963.5625$ kNm

**Step 2: Computation of $x_{u,lim}$**

Effective width of flange = $(l_o/6) + b_w + 6D_f = (12000/6) + 350 + 600 = 2,950$ mm.

$x_{u,max} = 0.48d = 0.48(660) = 316.80$ mm. This shows that the neutral axis is in the web of this beam.

$D_f/d = 100/660 = 0.1515 < 0.2$, and

$D_f/x_u = 100/316.8 = 0.316 < 0.43$

The expression of $M_{u,lim}$ is obtained from Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2) and is as follows:

$M_{u,lim} = 0.36(x_{u,max}/d)\{1 - 0.42(x_{u,max}/d)\} f_{ek} b_w d^2 + 0.45 f_{ek} (b_f - b_w) D_f (d - D_f/2)$

$= 0.36(0.48)\{1 - 0.42(0.48)\} (20)(350)(650)(650)$

$+ 0.45 (20) (2950 - 350) (100) (660 - 50) = 1,835.43$ kNm

The design moment $M_u = 963.5625$ kNm is less than $M_{u,lim}$. Hence, one under-reinforced beam can be designed.
Step 3: Determination of $x_u$

Since the design moment $M_u$ is almost 50% of $M_{u,\text{lim}}$, let us assume the neutral axis to be in the flange. The area of steel is to be calculated from the moment equation (Eq. 3.23 of Lesson 5), when steel is ensured to reach the design stress $f_y = 0.87 \times (415) = 361.05 \text{ N/mm}^2$. It is worth mentioning that the term $b$ of Eq. 3.23 of Lesson 5 is here $b_f$ as the $T$-beam is treated as a rectangular beam when the neutral axis is in the flange.

$$M_u = 0.87 \ f_y \ A_{st} \ d \left(1 - \frac{A_{st} f_y}{f_{ck} b \ d}\right)$$

(3.23)

Here, all but $A_{st}$ are known. However, this will give a quadratic equation of $A_{st}$ and the lower one of the two values will be provided in the beam. The above equation gives:

$$A_{st}^2 - 93831.3253 \ A_{st} + 379416711.3 = 0$$

which gives the lower value of $A_{st}$ as:

$$A_{st} = 4,234.722097 \text{ mm}^2$$. The reason of selecting the lower value of $A_{st}$ is explained in sec 3.6.4.8 of Lesson 6 in the solution of Design Problem 3.1.

Then, employing Eq. 3.16 of Lesson 5, we get

$$x_u = \frac{0.87 \ f_y \ A_{st}}{0.36 \ b \ f_{ck}}$$

(3.16)

or $x_u = 71.98 \text{ mm}$.  

Again, employing Eq. 3.24 of Lesson 5, we can determine $x_u$ first and then $A_{st}$ from Eq. 3.16 or 17 of Lesson 5, as explained in the next step.

Eq. 3.24 of Lesson 5 gives:

$$M_u = 0.36 \ (x_u/d) \ \{1 - 0.42 (x_u/d)\} \ f_{ck} \ b_f \ d^2$$

$$= 0.36 \ (x_u) \ \{1 - 0.42 \ (x_u/660)\} \ f_{ck} \ b_f \ d$$

$$963.5625 \times (10^6) = 0.36 \ (x_u) \ \{1 - 0.42 \ (x_u/660)\} \ (20) \ (2950) \ (660)$$

or $x_u = 72.03 \text{ mm}$.
The two values of $x_u$ are the same. It is thus seen that, the value of $x_u$ can be determined either first finding the value of $A_{st}$, from Eq. 3.23 of Lesson 5 or directly from Eq. 3.24 of Lesson 5 first and then the value of $A_{st}$ can be determined.

**Step 4: Determination of $A_{st}$**

Equating $C = T$, we have from Eq. 3.17 of Lesson 5:

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f d}$$

$$A_{st} = \frac{0.36 f_{ck} b_f x_u}{0.87 f_y} = \frac{0.36 \times (20) \times (2950) \times (72.03)}{0.87 \times (415)} = 4,237.41 \text{ mm}^2$$

Minimum $A_{st} = (0.85/f_y) b_w d = (0.85/415) \times (350) \times (660) = 473.13 \text{ mm}^2$

Maximum $A_{st} = 0.04 b_w D = (0.04) \times (350) \times (660) = 9,240 \text{ mm}^2$

Hence, $A_{st} = 4,237.41 \text{ mm}^2$ is o.k.

Provide 6 - 28 T (= 3694 mm$^2$) + 2-20 T (= 628 mm$^2$) to have total $A_{st} = 4,322 \text{ mm}^2$.

**Ex.6:** Design a beam in place of the beam of Ex.5 (Fig. 5.12.1) if the imposed loads are increased to 12 kN/m$^2$. Other data are: $D_f = 100 \text{ mm}$, $b_w = 350 \text{ mm}$, spacing of beams = 4000 mm c/c, effective span = 12 m simply supported and cover = 90 mm. Use Fe 415 and M 20.

**Solution:** As in Ex.5, $b_f = 2,950 \text{ mm}$.

**Step 1: Computation of factored bending moment**

Weight of slab/m$^2$ = 2.5 kN/m$^2$ (as in Ex.1)

Imposed loads = 12.0 kN/m$^2$ (given)

Total loads = 14.5 kN/m$^2$

Total weight of slab and imposed loads = 14.5 (4) = 58.0 kN/m

Dead loads of the beam = 0.65 (0.35) (25) = 5.6875 kN/m

Total loads = 63.6875 kN/m

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\[(M_u)_{\text{factored}} = \frac{1.5 \times (63.6875 \times 12 \times 12)}{8} = 1,719.5625 \text{ kNm}\]

**Step 2: Determination of** \(M_{u,\text{lim}}\)

\(M_{u,\text{lim}}\) of the beam of Ex.5 = 1,835.43 kNm. The factored moment of this problem (1,719.5625 kNm) is close to the value of \(M_{u,\text{lim}}\) of the section.

**Step 3: Determination of** \(d\)

Assuming \(D_f/d < 0.2\), we have from Eq. 5.7 of Lesson 10,

\[
M_u = 0.36 \left(\frac{x_{u,\text{max}}}{d}\right) \left(1 - 0.42 \left(\frac{x_{u,\text{max}}}{d}\right)\right) f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) D_f (d - D_f /2)
\]

\[
1719.5625 \times (10^6) = 0.36(0.48) \{1 - 0.42(0.48)\} (20) (350) d^2 + 0.45 (2600) (20) (100) (d - 50)
\]

Solving the above equation, we get \(d = 624.09\) mm, giving total depth = 624.09 + 90 = 715 mm (say).

Since the dead load of the beam is reduced due to decreasing the depth of the beam, the revised loads are calculated below:

Loads from the slab = 58.0 kN/m

Dead loads (revised) = 0.615 (0.35) (25) = 5.38125 kN/m

Total loads = 63.38125 kN/m

\[(M_u)_{\text{factored}} = \frac{1.5 \times (63.38125 \times 12 \times 12)}{8} = 1,711.29 \text{ kNm}\]

**Approximate value of** \(A_{st}\):

\[
A_{st} = \frac{M_u}{0.87 f_y (d - \frac{D_f}{2})} = \frac{1711.29 \times (10^6)}{0.87 (415) (625 - 50)} = 8,243.06 \text{ mm}^2
\]
Step 4: Determination of $A_{st}$ (Fig. 5.12.2)

\[x_u = x_{u,\text{max}} = 0.48 \times 625 = 300 \text{ mm}\]

Equating $T$ and $C$ (Eq. 5.5 of Lesson 10), we have:

\[0.87 f_y A_{st} = 0.36 x_{u,\text{max}} b_w f_{ck} + 0.45 f_{ck} (b_f - b_w) D_f\]

or

\[A_{st} = \frac{0.36 (300) (350) (20) + 0.45 (20) (2600) (100)}{0.87 (415)} = 8,574.98 \text{ mm}^2\]

Maximum $A_{st} = 0.04 b D = 0.04 (350) (715) = 10,010.00 \text{ mm}^2$

Minimum $A_{st} = \frac{(0.85/f_y) b_w d}{(0.85/415) (350) (625)} = 448.05 \text{ mm}^2$

Hence, $A_{st} = 8,574.98 \text{ mm}^2$ is o.k.

So, provide $8-36 \text{T} + 2-18 \text{T} = 8143 + 508 = 8,651 \text{ mm}^2$

Step 5: Determination of $x_u$

Using $A_{st} = 8,651 \text{ mm}^2$ in the expression of $T = C$ (Eq. 5.5 of Lesson 10), we have:

\[0.87 f_y A_{st} = 0.36 x_u b_w f_{ck} + 0.45 f_{ck} (b_f - b_w) D_f\]

or

\[x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck} (b_f - b_w)}{0.36 b_w f_{ck}}\]
So, \( A_{st} \) provided is reduced to \( 8-36 + 2-16 = 8143 + 402 = 8,545 \text{ mm}^2 \). Accordingly,

\[
x_u = \frac{0.87 \times (8545) - 0.45 \times (20) \times (2600) \times (100)}{0.36 \times (350) \times (20)} = 295.703 \text{ mm} < x_{u,max} (=300 \text{ mm})
\]

**Step 6: Checking of \( M_u \)**

\[
D_f/d = 100/625 = 0.16 < 0.2
\]

\[
D_f/x_u = 100/215.7 = 0.33 < 0.43. \text{ Hence, it is a problem of case (iii a) and } M_u \text{ can be obtained from Eq. 5.14 of Lesson 10.}
\]

So,

\[
M_u = 0.36(x_u/d) \{1 - 0.42(x_u/d)\} f_{ck} b_f d^2 + 0.45 f_{ck} (b_f - b_w) (D_f) (d - D_f/2)
\]

\[
= 0.36 \times (295.703/625) \{1 - 0.42 (295.703/625)\} \times (20) \times (350) \times (625) \times (625)
\]

\[
+ 0.45 \times (20) \times (2600) \times (100) \times (625 - 50)
\]

\[
= 1,718.68 \text{ kNm} > (M_{u,design} (=1,711.29 \text{ kNm})
\]

Hence, the design is o.k.

**Ex.7:** Determine the tensile reinforcement \( A_{st} \) of the flanged beam of Ex.5 (Fig. 5.12.1) when the imposed loads = 12 kN/m². All other parameters are the same as those of Ex.5: \( D_f = 100 \text{ mm}, \) \( D = 750 \text{ mm}, \) \( b_w = 350 \text{ mm}, \) spacing of beams = 4000 mm c/c, effective span = 12 m, simply supported, cover = 90 mm and \( d = 660 \text{ mm}. \) Use Fe 415 and M 20.

**Solution:**

**Step 1: Computation of factored bending moment \( M_u \)**

Dead loads of the slab (see Ex.5) = 2.5 kN/m²

Imposed loads = 12.0 kN/m²

Total loads = 14.5 kN/m²

Loads/m = 14.5 (4) = 58.0 kN/m

Dead loads of beam = 0.65 (0.35) (25) = 5.6875 kN/m
Total loads = 63.6875 kN/m

Factored $M_u = (1.5) (63.6875) (12) (12)/8 = 1,719.5625$ kNm.

**Step 2: Determination of $M_{u,lim}$**

From Ex.5, the $M_{u,lim}$ of this beam = 1,835.43 kNm. Hence, this beam shall be designed as under-reinforced.

**Step 3: Determination of $x_u$**

Assuming $x_u$ to be in the flange, we have from Eq. 3.24 of Lesson 5 and considering $b = b_f$,

$$M_u = 0.36 x_u \{1 - 0.42(x_u/d)\} f_{ck} b_f d$$

$$1719.5625 \times 10^6 = 0.36 x_u \{1 - 0.42(x_u/660)\} (20) (2950) (550)$$

Solving, we get $x_u = 134.1 > 100$ mm

So, let us assume that the neutral axis is in the web and $D_f/x_u < 0.43$, from Eq. 5.14 of Lesson 10 (case iii a of sec. 5.10.4.3), we have:

$$M_u = 0.36 (x_u/d) \{1 - 0.42(x_u/d)\} f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) (D_f) (d - D_f/2)$$

$$= 0.36 x_u \{1 - 0.42 (x_u/660)\} (20) (350) (660)$$

$$+ 0.45 (20) (2600) (100) (660 - 50)$$

Substituting the value of $M_u = 1,719.5625$ kNm in the above equation and simplifying,

$$x_u^2 - 1571.43 x_u + 276042 = 0$$

Solving, we have $x_u = 201.5$ mm

$$D_f/x_u = 100/201.5 = 0.496 > 0.43.$$  

So, we have to use Eq. 5.15 and 5.18 of Lesson 10 for $y_f$ and $M_u$ (case iii b of sec. 5.10.4.3). Thus, we have:

$$M_u = 0.36 x_u \{1 - 0.42(x_u/d)\} f_{ck} b_w d + 0.45 f_{ck} (b_f - b_w) y_f (d - y_f/2)$$

where, $y_f = (0.15 x_u + 0.65 D_f)$

So, $M_u = 0.36 x_u \{1 - 0.42 (x_u/660)\} (20) (350) (660)$
or \[1719.5625 \times 10^6 = 3.75165 \times 10^6 x_u - 795.15 x_u^2 + 954.4275 \times 10^6\]

Solving, we get \(x_u = 213.63\) mm.

\(D_f/x_u = 100/213.63 = 0.468 > 0.43.\)

Hence, o.k.

**Step 4: Determination of \(A_{st}\)**

Equating \(C = T\) from Eqs. 5.16 and 5.17 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:

\[0.87 f_y A_{st} = 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f\]

where, \(y_f = 0.15 x_u + 0.65 D_f\)

Here, using \(x_u = 213.63\) mm, \(D_f = 100\) mm, we get

\(y_f = 0.15 (213.63) + 0.65 (100) = 97.04\) mm

So,

\[A_{st} = \frac{0.36 (20) (350) (213.63) + 0.45 (20) (2600) (97.04)}{0.87 (415)} = 7,780.32 \text{ mm}^2\]

Minimum \(A_{st} = (0.85/f_y) (b_w) (d) = 0.85 (350) (660)/(415) = 473.13 \text{ mm}^2\)

Maximum \(A_{st} = 0.04 b_w D = 0.04 (350) (750) = 10,500 \text{ mm}^2\)

Hence, \(A_{st} = 7,780.32 \text{ mm}^2\) is o.k.
Provide 6-36 T + 3-28 T \ (6107 + 1847 = 7,954 \ \text{mm}^2). \text{ Please refer to Fig. 5.12.3.}

**Step 5: Checking of } x_u \text{ and } M_u \text{ using } A_{st} = 7,954 \ \text{mm}^2

From } T = C \text{ (Eqs. 5.16 and 5.17 of Lesson 10), we have

\[ 0.87 f_y A_{st} = 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f \]

where, \[ y_f = 0.15 x_u + 0.65 D_f \]

or \[ 0.87 \ (415) \ (7954) = 0.36 \ (20) \ (350) \ x_u + 0.45 \ (20) \ (2600) \ (0.15 \ x_u + 0.65 \ D_f) \]

or \[ x_u = 224.01 \ \text{mm} \]

\[ D_f / x_u = 100/224.01 = 0.446 > 0.43. \text{ Accordingly, employing Eq. 5.18 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:} \]

\[ M_u = 0.36 \ x_u \ (1 - 0.42( x_u/d)) \ f_{ck} b_w d + 0.45 \ f_{ck} (b_f - b_w) \ y_f (d - y_f/2) \]

\[ = 0.36 \ (224.01) \ (1 - 0.42 (224.01/660)) \ (20) \ (350) \ (660) \]

\[ + 0.45 \ (20) \ (2600) \ [(0.15) 224.01 + 65] \ (660) - 0.15 (112) - 32.5} \]

\[ = 1,779.439 \ \text{kNm} > 1,719.5625 \ \text{kNm} \]

Hence, o.k.
Ex.8: Design the flanged beam of Fig. 5.12.4, given in following: \( D_f = 100 \text{ mm} \), \( D = 675 \text{ mm} \), \( b_w = 350 \text{ mm} \), spacing of beams = 4000 mm c/c, effective span = 12 m simply supported, cover = 90 mm, \( d = 585 \text{ mm} \) and imposed loads = 12 kN/m\(^2\). Use Fe 415 and M 20.

Step 1: Computation of factored bending moment, \( M_u \)

Weight of slab/m\(^2\) = \((0.1) (25) = 2.5 \text{ kN/m}^2\)

Imposed loads = 12.0 kN/m\(^2\)

Total loads = 14.5 kN/m\(^2\)

Total weight of slab + imposed loads/m = 14.5 (4) = 58 kN/m

Dead loads of beam = 0.575 (0.35) (25) = 5.032 kN/m

Total loads = 63.032 kN/m

Factored \( M_u = (1.5) (63.032) (12) (12)/8 = 1,701.87 \text{ kNm} \)

Step 2: Determination of \( M_{u,\text{lim}} \)

Assuming the neutral axis to be in the web, \( D_f/x_u < 0.43 \) and \( D_f/d = 100/585 \approx 0.17 < 0.2 \), we consider the case (ii a) of sec. 5.10.4.2 of Lesson 10 to get the following:

\[
M_{u,\text{lim}} = 0.36 \frac{x_{u,\text{max}}}{d} \left(1 - 0.42 \frac{x_{u,\text{max}}}{d}\right) f_{ck} b_w d^2 \\
+ 0.45 f_{ck} (b_f - b_w) D_f (d - D_f/2) \\
= 0.36(0.48) \left(1 - 0.42 (0.48)\right) (20) (350) (585) (585) \\
+ 0.45(20) (2600) (100) (585 - 50) = 1,582.4 \text{ kNm}
\]

Since, factored \( M_u > M_{u,\text{lim}} \), the beam is designed as doubly reinforced.

\[
M_{u,2} = M_u - M_{u,\text{lim}} = 1701.87 - 1582.4 = 119.47 \text{ kNm}
\]
Step 3: Determination of area of steel

\[ A_{st,\text{lim}} \text{ is obtained equating } T = C \text{ (Eqs. 5.5 and 6 of Lesson 10).} \]

\[ 0.87 \, f_y \, (A_{st,\text{lim}}) = 0.36 \, b_w \, (x_{u,\text{max}} / d) \, d \, f_{ck} + 0.45 \, f_{ck} \, (b_f - b_w) \, D_f \]

or

\[ A_{st,\text{lim}} = \frac{0.36 \, b_w \, (x_{u,\text{max}} / d) \, d \, f_{ck} + 0.45 \, f_{ck} \, (b_f - b_w) \, D_f}{0.87 \, f_y} \]

\[ = \frac{0.36 \, (350) \, (0.48) \, (585) \, (20) + 0.45 \, (20) \, (2600) \, (100)}{0.87 \, (415)} = 8,440.98 \text{ mm}^2 \]

\[ A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc}) \, (d - d')} \quad \text{(Eq. 4.4 of Lesson 8).} \]

where

\[ f_{sc} = 353 \text{ N/mm}^2 \text{ for } d'/d = 0.1 \]

\[ f_{cc} = 0.446 \, f_{ck} = 0.446 \, (20) = 8.92 \text{ N/mm}^2 \]

\[ M_{u2} = 119.47 \times 10^6 \text{ Nmm} \]

\[ d' = 58.5 \text{ mm} \]

\[ d = 585 \text{ mm} \]
Using the above values in the expression of $A_{sc}$ (Eq. 4.4 of Lesson 8), we get

$$A_{sc} = 659.63 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y}$$

(Eqs. 4.4 and 4.5 of Lesson 8).

Substituting the values of $A_{sc}$, $f_{sc}$, $f_{cc}$ and $f_y$ we get

$$A_{st2} = 628.48 \text{ mm}^2$$

Total $A_{st} = A_{st,lim} + A_{st2} = 8,440.98 + 628.48 = 9,069.46 \text{ mm}^2$

Maximum $A_{st} = 0.04 b_w D = 0.04 (350) (675) = 9,450 \text{ mm}^2$

and minimum $A_{st} = (0.85/f_y) b_w d = (0.85/415) (350) (585) = 419.37 \text{ mm}^2$

Hence, $A_{st} = 9,069.46 \text{ mm}^2$ is o.k.

Provide 8-36 T + 3-20 T = 8143 + 942 = 9,085 mm² for $A_{st}$ and 1-20 + 2-16 = 314 + 402 = 716 mm² for $A_{sc}$ (Fig. 5.12.5).

**Step 4: To check for $x_u$ and $M_u$ (Fig. 5.12.5)**

Assuming $x_u$ in the web and $D_f/x_u < 0.43$ and using $T = C$ (case ii a of sec. 5.10.4.2 of Lesson 10 with additional compression force due to compression steel), we have:

$$0.87 f_y A_{st} = 0.36 b_w x_u f_{ck} + 0.45 (b_f - b_w) f_{ck} D_f + A_{sc} (f_{sc} - f_{cc})$$

or

$$0.87 (415) (9085) = 0.36 (350) x_u (20) + 0.45 (2600) (20) (100) + 716 \{353 - 0.45 (20)\}$$

This gives $x_u = 275.33 \text{ mm}$.

$$x_{u,max} = 0.48 (d) = 0.48 (585) = 280.8 \text{ mm}.$$  

So, $x_u < x_{u,max}$, $D_f/x_u = 100/275.33 = 0.363 < 0.43$

and $D_f/d = 100/585 = 0.17 < 0.2$.

The assumptions, therefore, are correct. So, $M_u$ can be obtained from Eq. 5.14 of sec. 5.10.4.3 of Lesson 10 with additional moment due to compression steel, as given below:
So, \[ M_u = 0.36 \, b_w \, x_u \, f_{ck} \, (d - 0.42 \, x_u) + 0.45 \, (b_t - b_w) \, f_{ck} \, D_t \, (d - D_t/2) \]

\[ + \, A_{sc} \, (f_{sc} - f_{cc}) \, (d - d') \]

\[ = 0.36 \, (350) \, (275.33) \, (20) \, \{585 - 0.42 \, (275.33)\} \]

\[ + \, 0.45 \, (2600) \, (20) \, (100) \, (585 - 50) + 716 \, (344) \, (585 - 58.5) \]

\[ = 325.66 + 1251.9 + 129.67 = 1707.23 \text{ kNm} \]

Factored moment = 1701.87 kNm < 1707.23 kNm. Hence, o.k.

5.12.4 Practice Questions and Problems with Answers

Q.1: Determine the steel reinforcement of a simply supported flanged beam (Fig. 5.12.6) of \( D_t = 100 \text{ mm}, \) \( D = 700 \text{ mm}, \) cover = 50 mm, \( d = 650 \text{ mm}, \) \( b_w = 300 \text{ mm}, \) spacing of the beams = 4,000 mm c/c, effective span = 10 m and imposed loads = 10 kN/m². Use M 20 and Fe 415.

A.1: Solution:

Step 1: Computation of \( (M_u)_{\text{factored}} \)

Weight of slab = \( (0.1) \, (25) = 2.5 \text{ kN/m}^2 \)

Imposed loads = \[ 10.0 \text{ kN/m}^2 \]

\[ 12.5 \text{ kN/m}^2 \]
Total loads per m = (12.5) (4) = 50 kN/m

Dead loads of beam = (0.3) (0.6) (25) = 4.50 kN/m

Total loads = 54.50 kN/m

Factored \( M_u = (1.5) (54.50) (10) (10)/8 = 1,021.87 \text{ kNm} \)

**Step 2: Determination of** \( M_{u,lim} \)

Effective width of the flange \( b_f = l_o/6 + b_w + 6 D_f = (10,000/6) + 300 + 600 = 2,567 \text{ mm.} \)

\[ x_{u,\text{max}} = 0.48 \ d = 0.48 \ (650) = 312 \text{ mm} \]

Hence, the balanced neutral axis is in the web of the beam.

\[ D_f/d = 100/650 = 0.154 < 0.2 \]

\[ D_f/x_u = 100/312 = 0.32 < 0.43 \]

So, the full depth of flange is having a stress of 0.446 \( f_{ck} \). From Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2), we have,

\[
M_{u,lim} = 0.36 \left( \frac{x_{u,\text{max}}}{d} \right) \left\{ 1 - 0.42 \left( \frac{x_{u,\text{max}}}{d} \right) \right\} f_{ck} \ b_w \ d^2 \]

\[
+ 0.45 \ f_{ck} \ (b_f - b_w) \ D_f (d - D_f/2) \]

\[
= 0.36(0.48) \left\{ 1 - 0.42(0.48) \right\} (20) (300) (650) (650) \]

\[
+ 0.45(20) (2267) (100) (650 - 50) \]

\[
= 1573.92 \text{ kNm} > M_u (= 1021.87 \text{ kNm}) \]

So, the beam will be under-reinforced one.

**Step 3: Determination of** \( x_u \)

Assuming \( x_u \) is in the flange, we have from Eq. 3.24 of Lesson 5 (rectangular beam when \( b = b_f \)).

\[
M_u = 0.36 \left( \frac{x_u}{d} \right) \left\{ 1 - 0.42 \left( \frac{x_u}{d} \right) \right\} f_{ck} \ b_f \ d^2 \]

\[
= 0.36 \ x_u \left\{ 1 - 0.42 \left( \frac{x_u}{d} \right) \right\} f_{ck} \ b_f \ d \]

\[
1021.87 \ (10^6) = 0.36 \ x_u \left\{ 1 - 0.42(x_u/650) \right\} (20) (2567) (650) \]

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\[ x_u = 89.55 \text{ mm}^2 < 100 \text{ mm} \] (Hence, the neutral axis is in the flange.)

**Step 4: Determination of \( A_{st} \)**

Equating \( C = T \), we have from Eq. 3.17 of Lesson 5:

\[
\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f d}
\]

or

\[
A_{st} = \frac{0.36 f_{ck} b_f x_u}{0.87 f_y} = \frac{0.36 (20) (2567) (89.55)}{0.87 (415)} = 4,584.12 \text{ mm}^2
\]

Minimum \( A_{st} = (0.85/f_y) (b_w) d = \frac{0.85 (300) (650)}{415} = 399.39 \text{ mm}^2 \)

Maximum \( A_{st} = 0.04 b_w D = (0.04) (300) (700) = 8,400 \text{ mm}^2 \)

So, \( A_{st} = 4,584.12 \text{ mm}^2 \) is o.k.

Provide \( 6-28 \text{T} + 2-25 \text{T} = 3694 + 981 = 4,675 \text{ mm}^2 \) (Fig. 5.12.6).

**5.12.5 References**


**5.12.6 Test 12 with Solutions**

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

![Diagram](image.png)

**TQ.1:** Determine the steel reinforcement $A_{st}$ of the simply supported flanged beam of Q.1 (Fig. 5.12.6) having $D_f = 100$ mm, $D = 700$ mm, cover = 50 mm, $d = 650$ mm, $b_w = 300$ mm, spacing of the beams = 4,000 mm c/c, effective span = 12 m and imposed loads = 10 kN/m². Use M 20 and Fe 415.

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A.TQ.1: Solution:

Step 1: Computation of \((M_u)\) factored

Total loads from Q.1 of sec. 5.12.4 = 54.50 kN/m

Factored \(M_u = (1.5)(54.50)(12)(12)/8 = 1,471.5\) kNm

Step 2: Determination of \(M_{u,lim}\)

Effective width of flange \(= l_o/6 + b_w + 6 D_f\)

\[= (12000/6) + 300 + 600 = 2,900\text{ mm} \text{ (Fig. 5.12.7)}\]

\(x_{u,max} = 0.48d = 0.48(650) = 312\) mm

Hence, the balanced neutral axis is in the web.

\(D_f/d = 100/650 = 0.154 < 0.2\)

\(D_f/x_u = 100/312 = 0.32 < 0.43\)

So, the full depth of flange is having constant stress of 0.446 \(f_{ck}\). From Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2), we have

\[M_{u,lim} = 0.36 \left(\frac{x_{u,max}}{d}\right)\{1 - 0.42 \left(\frac{x_{u,max}}{d}\right)\} f_{ck} b_w d^2\]

\[+ 0.45 f_{ck} (b_f - b_w) D_f (d - D_f/2)\]

\[= 0.36(0.48) \{1 - 0.42(0.48)\} (20)(300)(650)(650)\]

\[+ 0.45(20)(2600)(100)(650 - 50) = 1,753.74 \text{ kNm} > 1,471.5\]

kNm

So, the beam will be under-reinforced.

Step 3: Determination of \(x_u\)

Assuming \(x_u\) to be in the flange, we have from Eq. 3.24 of Lesson 5 (singly reinforced rectangular beam when \(b = b_f\)):

\[M_u = 0.36 x_u \{1 - 0.42 (x_u/d)\} f_{ck} b_f d\]

or \(1471.5 \times 10^6 = 0.36 (x_u) \{1 - 0.42 (x_u/650)\} (20)(2900)(650)\)

or \(x_u^2 - 1547.49 x_u + 167.81 \times 10^3 = 0\)

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Solving, we have \( x_u = 117.34 \text{ mm} > 100 \text{ mm} \)

So, neutral axis is in the web.

Assuming \( D_f/x_u < 0.43 \), we have from Eq. 5.14 of Lesson 10 (case iii a of sec. 5.10.4.3),

\[
M_u = 0.36 x_u \{1 - 0.42 (x_u/d)\} f_{ck} b_w d + 0.45 f_{ck} (b_f - b_w) D_f (d - D_f/2)
\]

\[
= 0.36 x_u \{1 - 0.42 (x_u/650)\} (20) (300) (650)
\]

\[
+ 0.45(20) (2600) (100) (650 - 50)
\]

or \( x_u^2 - 1547.62 x_u + 74404.7 = 0 \)

Solving, we have \( x_u = 49.67 < 100 \text{ mm} \)

However, in the above when it is assumed that the neutral axis is in the flange \( x_u \) is found to be 117.34 mm and in the second trial when \( x_u \) is assumed in the web \( x_u \) is seen to be 49.67 mm. This indicates that the full depth of the flange will not have the strain of 0.002, neutral axis is in the web and \( D_f/x_u \) is more than 0.43. So, we have to use Eq. 5.18 of Lesson 10, with the introduction of \( y_f \) from Eq. 5.15 of Lesson 10.

Assuming \( D_f/x_u > 0.43 \), from Eqs. 5.15 and 5.18 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:

\[
M_u = 0.36 x_u \{1 - 0.42 (x_u/d)\} f_{ck} b_w d + 0.45 f_{ck} (b_f - b_w) y_f (d - y_f/2)
\]

where, \( y_f = (0.15 x_u + 0.65 D_f) \)

So, \( M_u = 0.36 x_u \{1 - 0.42 (x_u/650)\} (20) (300) (650) \)

\[
+ 0.45(20) (2600) (0.15 x_u + 0.65) (650 - 0.075 x_u - 0.325 x_u)
\]

or, \( 1471.5 \times 10^6 = -1170.45 x_u^2 + 3.45735 x_u + 939.2175 \times 10^6 \)

Solving, we get \( x_u = 162.9454 \text{ mm} \). This shows that the assumption of \( D_f/x_u > 0.43 \) is correct as \( D_f/x_u = 100 / 162.9454 = 0.614 \).

**Step 4: Determination of \( A_{st} \)**

Equating \( C = T \) from Eqs. 5.16 and 5.17 of Lesson 10 (case iii b of sec. 5.10.4.3), we have
\[ 0.87 f_y A_{st} = 0.36 f_{ck} b_w x_w + 0.45 f_{ck} (b_f - b_w) y_f \]

or
\[ A_{st} = \frac{0.36(20)(30)(162.9454) + 0.45(20)(2600)(0.15(162.9454) + 65)}{0.87(415)} \]
\[ = 974.829 + 5,796.81 = 6,771.639 \text{ mm}^2 \]

Minimum \[ A_{st} = (0.85/f_y)(b_w)(d) = 0.85(300)(650)/415 = 399.39 \text{ mm}^2 \]

Maximum \[ A_{st} = 0.04(b_w)(D) = 0.04(300)(700) = 8,400 \text{ mm}^2 \]

So, \[ A_{st} = 6,771.639 \text{ is o.k.} \]

Provide \[ 2\text{-}36T + 6\text{-}32T = 2035 + 4825 = 6,860 \text{ mm}^2 > 6,771.639 \text{ mm}^2 \]
(Fig. 5.12.7).

5.12.7 Summary of this Lesson

This lesson explains the steps involved in solving the design type of numerical problems. Further, several examples of design type of numerical problems are illustrated explaining the steps of their solutions. Solutions of practice problems and test problems will give the readers confidence in applying the theory explained in Lesson 10 in solving the numerical problems.