CRYOGENIC ENGINEERING

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Lecture No - 16
• In the earlier lecture, we have seen a Claude system, in which the energy content in the gas is removed by allowing it to do some work in an expansion device. $y$ and $W/m$ are given by

$$y = \left( \frac{h_1 - h_2}{h_1 - h_f} \right) + x \left( \frac{h_3 - h_e}{h_1 - h_f} \right)$$

$$-\frac{W_{\text{net}}}{\dot{m}} = \begin{cases} \left( T_1(s_1 - s_2) - (h_1 - h_2) \right) \\ -x(h_3 - h_e) \end{cases}$$
Earlier Lecture

• **Liquid yield v/s. x**

• In a reversible Claude system, if $T_1, T_2, T_3$ are held constant

  - The yield $y$ goes through a maxima with the increase in the value of $x$.
  
  - Also, this maxima shifts to the right and decreases with the decrease in $T_3$.
Earlier Lecture

- \( \frac{W}{m_f} \) v/s. \( x \)

In a reversible Claude system, if \( T_1, T_2, T_3 \) are held constant:

- \( \frac{W}{m_f} \) of the system goes through a minima with an increase in \( x \).

- Also, the position of the minima shifts to the right and increases with the decrease in the value of \( T_3 \).
Outline of the Lecture

Topic: Gas Liquefaction and Refrigeration Systems (contd)

• Claude System with irreversibilities in Compressor and Expander

• Kapitza System

• Heylandt System

• Collins System
  • Liquid yield
  • Work requirement
Introduction

• The compression and expansion processes in an actual Claude cycle are irreversible.

• These irreversibilities cause inefficiencies and deteriorate the performance of the system.

• To study the effect of these inefficiencies, a tutorial problem is solved.

• The results are graphically plotted and compared with a reversible system solved in the previous lecture.
The T – s diagram for a reversible Claude system is as shown.

- The compressor irreversibility is shown by the process \(1 \rightarrow 2'\).
- Similarly, the expander irreversibility is denoted by the process \(3 \rightarrow e'\).
Claude System

- The compressor inefficiency is due to both frictional losses ($\eta_{mech,c}$) and non-isothermal process ($\eta_{iso,c}$).

- The net irreversibility is given by $\eta_{oval,c} = \eta_{mech,c} \times \eta_{iso,c}$

- Similarly, the expander inefficiency is due to both frictional losses ($\eta_{mech,e}$) and non-isentropic process ($\eta_{ad,e}$).

- The net irreversibility is given by $\eta_{oval,e} = \eta_{mech,e} \times \eta_{ad,e}$
Claude System

- With these inefficiencies taken into account, the yield of the system decreases and the work requirement increases.

- The yield and work requirement of the system are given by

\[
y = \left( \frac{h_1 - h_2}{h_1 - h_f} \right) + x \left( \eta_{ad,e} \right) \left( \frac{h_3 - h_e}{h_1 - h_f} \right)
\]

\[
\frac{W_{net}}{\dot{m}} = \frac{\left( T_1 \left( s_1 - s_2 \right) - (h_1 - h_2) \right)}{\eta_{oval,e}} - x \left( \eta_{oval,e} \right) (h_3 - h_e)
\]
Tutorial

A. Determine $W/m_f$ for a Claude Cycle with $N_2$ as working fluid. The system operates between 1.013 bar (1 atm) and 50.65 bar (50 atm). The expander inlet $T_3$ is at 250 K. The expander flow ratio is varied between 0.1 and 0.9. The efficiencies are as given below.

<table>
<thead>
<tr>
<th>Comp.</th>
<th>$\eta_{oval,c} = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expd.</td>
<td>$\eta_{mech,e} = 0.86$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{ad,e} = 0.86$</td>
</tr>
</tbody>
</table>

B. Repeat the above problem for $T_3 = 300$ K, 275 K and 250 K. Plot the data $y$, $W/m_f$ versus $x$ graphically and comment on the results.
For above System, Calculate

1 Work/unit mass of gas liquefied

<table>
<thead>
<tr>
<th>N₂</th>
<th>Point 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>300 K</td>
</tr>
<tr>
<td>II</td>
<td>275 K</td>
</tr>
<tr>
<td>III</td>
<td>250 K</td>
</tr>
</tbody>
</table>
Methodology

• In the earlier lecture, an assignment problem on a reversible Claude cycle with the answers was given.

• As stated earlier, the same problem is taken up and the effects of inefficiencies of the compressor and the expander are studied.

• All the calculations are left as an exercise for the students and the final results are graphically plotted.
Tutorial

• **Liquid yield v/s. x**

The plot for $y$ v/s $x$ for a $T_3 = 300$ and $275$ K is shown.

- It is clear that maximum yield of the system decreases due to the irreversibility.

- The % decrease in the $y_{max}$ is 10% and 9% for $300$ and $275$ K respectively.
The plot for $W/m_f$ v/s $x$ for a $T_3 = 300$ and $275$ K is shown.

It is clear that minimum work requirement of the system increases due to the irreversibility.

The % increase in the $W/m_{f_{min}}$ is 89% and 87% for 300 and 275 K respectively.

Claude System
$N_2$, 50 atm

$\eta_{oval,c} = 0.75$
$\eta_{mech,e} = 0.86$
$\eta_{ad,e} = 0.86$

$\eta_{oval,c} = 1$
$\eta_{mech,e} = 1$
$\eta_{ad,e} = 1$
Kapitza & Heylandt System

• The transportation of gases across the world is done in liquid state by storing them at cryogenic temperatures.

• The air liquefaction is of primary importance because LN$_2$ and LOX are separated from LAir.

• Kapitza and Heylandt systems are the two different modifications of the Claude System which are generally used in the air liquefaction.

• Collins system, also a modification of Claude system, is widely used in liquefaction of Helium.
Kapitza System

• A Kapitza system is a low – pressure system which is used in Air liquefaction.

• It was invented in 1939 by Pyotr Kapitza, in which
  • The first heat exchanger is replaced by a set of valved regenerators.
  • The third heat exchanger is eliminated in the Claude system.
The regenerator/heat exchanger performs two different operations:
- Gas cooling/warming
- Gas purification

During one cycle, one unit purifies by freezing the impurities and cools the incoming hot gas.
Kapitza System

- While the other unit warms the outgoing gas and simultaneously removes the frozen impurities by evaporation.

- The valve mechanism is used to periodically change over from one unit to another (not shown in the figure).
Kapitza System

- This periodic alternation of units along with the counter – blow arrangement ensures a continuous performance.
- This system was the first one to use a turbo – expander (rotary type) instead of a reciprocating expander.
- This modification allowed the elimination of third heat exchanger in Claude system.
• The yield and work requirement of the system are given by the following equations.

\[ y = \left( \frac{h_1 - h_2}{h_1 - h_f} \right) + x \left( \frac{h_3 - h_e}{h_1 - h_f} \right) \]

\[ \frac{W_{net}}{m} = \begin{cases} 
(T_1 (s_1 - s_2) - (h_1 - h_2)) \\
-x(h_3 - h_e) 
\end{cases} \]

• Where, the expander mass flow ratio is denoted by \( x \).
Heylandt System

- Heylandt System is a high-pressure system, which is used in Air liquefaction.

- The typical operating pressure is around 200 atm.

- In 1949, Heylandt observed that, when a Claude system operated on Air with 200 atm and $x=0.6$, the optimum value of $T_3$ before the expansion engine is close to ambient.
Heylandt System

- He then eliminated the first heat exchanger.

- This modified system is called as Heylandt system.

- In this system, the inlet to the expander is at ambient and hence, the lubrication on the high pressure side and the operation of the expander are greatly simplified.
Heylandt System

- The yield and work requirement of the system are given by the following equations.

\[
y = \left( \frac{h_1 - h_2}{h_1 - h_f} \right) + x \left( \frac{h_2 - h_e}{h_1 - h_f} \right) \\
\]

\[
- \frac{W_{\text{net}}}{\dot{m}} = \begin{cases} 
(T_1 (s_1 - s_2) - (h_1 - h_2)) \\
x(h_2 - h_e) 
\end{cases}
\]

- Where, the expander mass flow ratio is denoted by \(x\).
The schematic of the Collins System is as shown. It was invented in the year 1946 by Samuel C. Collins at MIT, USA. This system is considered as one of the biggest milestones in Cryogenic Engineering.
Collins System

- This system is an extension to the Claude System.

- The system has a compressor, a J – T expansion device, a make up gas connection, five 2 – fluid heat exchangers and two turbo – expanders.

- Depending on the helium inlet pressure, two to six expansion devices are used.
Expansion engines are used to remove the heat from the gas and thereby to reach lower and lower temperatures.

The inversion temperature of Helium is around 45 K and in order to have a yield, $T_7$ should be less than 7.5 K.

Depending upon the mass flow rates, two to six expanders are used.
Collins System
Collins System

• Consider a control volume as shown in the figure.

<table>
<thead>
<tr>
<th>IN</th>
<th>OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>m @ 2</td>
<td>(W_{e1})</td>
</tr>
<tr>
<td></td>
<td>(W_{e2})</td>
</tr>
<tr>
<td></td>
<td>(m - m_f) @ 1</td>
</tr>
<tr>
<td></td>
<td>(m_f) @ f</td>
</tr>
</tbody>
</table>

• Applying 1\(^{st}\) Law, we have

\[
\dot{m}h_2 = W_{e1} + W_{e2} + (\dot{m} - \dot{m}_f)h_1 + \dot{m}_f h_f
\]
Let the work done by each of the expander be

\[ W_{e1} = \dot{m}_{e1} (\Delta h_1) \quad \text{and} \quad W_{e2} = \dot{m}_{e2} (\Delta h_2) \]

\( \Delta h_1 \) and \( \Delta h_2 \) are the enthalpy drops across the expander 1 and 2 respectively.

Substituting, we get

\[ \dot{m}h_2 = \left\{ \dot{m}_{e1} (\Delta h_1) + \dot{m}_{e2} (\Delta h_2) \right\} + \left( \dot{m} - \dot{m}_f \right) h_1 + \dot{m}_f h_f \]
Rearranging, we have

\[
\begin{bmatrix}
12 \\
12
\end{bmatrix}
\begin{bmatrix}
\Delta f \\
\Delta h
\end{bmatrix}
= 
\begin{bmatrix}
11 \\
11
\end{bmatrix}
\begin{bmatrix}
\Delta f \\
\Delta h
\end{bmatrix}
- 
\begin{bmatrix}
\Delta f \\
\Delta h
\end{bmatrix}
\]

The 1\textsuperscript{st} term is the yield for a simple L – H system.

The 2\textsuperscript{nd} term is the change in the yield occurring due to the modification.
For a given initial and final conditions of $p$, the yield $y$ depends on $h_3(T_3)$, $h_5(T_5)$, $x_1$ and $x_2$.

Like in the Claude system, the values of $T_3$, $T_5$, $x_1$ and $x_2$ have to optimized to obtain a maximum yield.
Collins System

- As stated earlier, using a control volume, 1\textsuperscript{st} and 2\textsuperscript{nd} Laws for a compressor, we get
  \[ -W_c = \dot{m} \left( T_1 (s_1 - s_2) - (h_1 - h_2) \right) \]

- Similarly, the control volume for an expansion engines, we get
  \[ W_{e1} = \dot{m}_{e1} (\Delta h_1) \quad W_{e2} = \dot{m}_{e2} (\Delta h_2) \]

- The net work done is given by
  \[ \therefore \quad \frac{-W_{net}}{\dot{m}} = -\frac{W_c}{\dot{m}} - \frac{W_{e1}}{\dot{m}} - \frac{W_{e2}}{\dot{m}} \]
Substituting, we have

\[-W_{net} = \frac{\left(T_1 (s_1 - s_2) - (h_1 - h_2)\right)}{\dot{m}} - x_1 (\Delta h_1) - x_2 (\Delta h_2)\]

- The 1st term is the work requirement for a simple L – H system.
- The 2nd term is the reduction in work requirement occurring due to the modification.
Tutorial

- Determine $y$, $W/m_f$, FOM for a Collins System with Helium as working fluid. The system operates between 1.013 bar (1 atm) and 15.19 bar (15 atm). The expander flow ratios are $x_1=0.6$, $x_2=0.2$ respectively. The expander inlet conditions are as mentioned below.

<table>
<thead>
<tr>
<th>Exp. Inlet Cond.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I</strong></td>
<td>60 K, 15 atm</td>
</tr>
<tr>
<td><strong>II</strong></td>
<td>15 K, 15 atm</td>
</tr>
</tbody>
</table>
## Tutorial

### Given

<table>
<thead>
<tr>
<th>Cycle: Collins System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working Pressure: 1 atm → 15 atm</td>
</tr>
<tr>
<td>Working Fluid: Helium</td>
</tr>
<tr>
<td>Expander 1: 15 atm, 60 K, ( x_1 = 0.4 )</td>
</tr>
<tr>
<td>Expander 2: 15 atm, 15 K, ( x_2 = 0.2 )</td>
</tr>
</tbody>
</table>

### For above System, Calculate

<table>
<thead>
<tr>
<th></th>
<th>Work/unit mass of gas liquefied</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>FOM</td>
</tr>
</tbody>
</table>
### Tutorial

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p (bar)</td>
<td>1.013</td>
<td>15.19</td>
<td>15.19</td>
</tr>
<tr>
<td>T (K)</td>
<td>300</td>
<td>300</td>
<td>60</td>
</tr>
<tr>
<td>h (J/g)</td>
<td>1587</td>
<td>1570</td>
<td>328</td>
</tr>
<tr>
<td>s (J/gK)</td>
<td>31.5</td>
<td>25.6</td>
<td>17.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>5</th>
<th>e₁</th>
<th>e₂</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>p (bar)</td>
<td>15.19</td>
<td>1.013</td>
<td>1.013</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>T (K)</td>
<td>15</td>
<td>22</td>
<td>4.8</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>h (J/g)</td>
<td>81</td>
<td>130.0</td>
<td>38</td>
<td>9.5</td>
<td></td>
</tr>
<tr>
<td>s (J/gK)</td>
<td>9.25</td>
<td>17.5</td>
<td>9.25</td>
<td>3.4</td>
<td></td>
</tr>
</tbody>
</table>

* Points e₁ and e₂ are located on p=1bar line by drawing vertical lines from point 3 and 5.
• The T – s diagram for a Collins System is as shown (not to scale).

• The expander inlet conditions are
  • 60 K
  • 15 K
Tutorial

• Liquid yield

\[ y = \left( \frac{h_1 - h_2}{h_1 - h_f} \right) + x_1 \left( \frac{h_3 - h_{e1}}{h_1 - h_f} \right) + x_2 \left( \frac{h_5 - h_{e2}}{h_1 - h_f} \right) \]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>e_1</th>
<th>e_2</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>1.013</td>
<td>15.19</td>
<td>15.19</td>
<td>15.19</td>
<td>1.013</td>
<td>1.013</td>
<td>1.01</td>
</tr>
<tr>
<td>T</td>
<td>300</td>
<td>300</td>
<td>60</td>
<td>15</td>
<td>22</td>
<td>4.8</td>
<td>4.2</td>
</tr>
<tr>
<td>h</td>
<td>1587</td>
<td>1570</td>
<td>328</td>
<td>81</td>
<td>130.0</td>
<td>38</td>
<td>9.5</td>
</tr>
<tr>
<td>s</td>
<td>31.5</td>
<td>25.6</td>
<td>17.5</td>
<td>9.25</td>
<td>17.5</td>
<td>9.25</td>
<td>3.4</td>
</tr>
</tbody>
</table>

\[ y = \frac{(1587 - 1570)}{(1587 - 9.5)} + 0.4 \frac{(328 - 130.0)}{(1587 - 9.5)} + 0.2 \frac{(81 - 38)}{(1587 - 9.5)} = 0.066 \]
Tutorial

- Work/unit mass of He compressed

\[ \frac{-W_{net}}{m} = \left( T_1 (s_1 - s_2) - (h_1 - h_2) \right) - x_1 (h_3 - h_{e1}) - x_2 (h_5 - h_{e2}) \]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>e_1</th>
<th>e_2</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>1.013</td>
<td>15.19</td>
<td>15.19</td>
<td>15.19</td>
<td>1.013</td>
<td>1.013</td>
<td>1.01</td>
</tr>
<tr>
<td>T</td>
<td>300</td>
<td>300</td>
<td>60</td>
<td>15</td>
<td>22</td>
<td>4.8</td>
<td>4.2</td>
</tr>
<tr>
<td>h</td>
<td>1587</td>
<td>1570</td>
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<td>9.25</td>
<td>17.5</td>
<td>9.25</td>
<td>3.4</td>
</tr>
</tbody>
</table>

\[ \frac{-W_{net}}{m} = \begin{cases} 300(31.5 - 25.6) - (1587 - 1570) \\ -0.4(328 - 130.0) - 0.2(81 - 38) \end{cases} = 1665.2 J / g \]
Tutorial

• Work/unit mass of He liquefied

\[-\frac{W_{\text{net}}}{\dot{m}} = 1665.2\]

\[y = 0.066\]

\[-\frac{W_{\text{net}}}{\dot{m}_f} = -\frac{W_{\text{net}}}{y\dot{m}} = \frac{1665.2}{0.066} = 25230.3 \text{ J/g}\]

• Figure of Merit (FOM)

\[-\frac{W_{\text{net}}}{\dot{m}_f} = 25230.3\]

\[-\frac{W_i}{\dot{m}_f} = 6837\]

\[FOM = \frac{W_i}{\dot{m}_f} / \frac{W_{\text{net}}}{\dot{m}_f} = \frac{6837}{25230.3} = 0.271\]
Summary

• The compression and expansion processes in an actual Claude cycle are irreversible. These cause inefficiencies and deteriorate the performance of the system.

• Kapitza and Heylandt systems are the two modifications of the Claude System.

• In a Kapitza cycle, the regenerator/heat exchanger performs both gas cooling/warming and gas purification.
Summary

• Also, it was first system to use a turbo – expander (rotary type) instead of a reciprocating expander.

• Heylandt System is a high – pressure system, which is used in Air liquefaction (~200 atm).

• In this system, the inlet to the expander is ambient and hence, the lubrication on the high pressure side and the operation of the expander is greatly simplified.
Summary

• The Collins system is an extension to the Claude System and depending on the helium inlet pressure, two to six expansion devices are used.

• The yield and work requirement are given by

\[
y = \left( \frac{h_1 - h_2}{h_1 - h_f} \right) + x_1 \left( \frac{h_3 - h_{e1}}{h_1 - h_f} \right) + x_2 \left( \frac{h_5 - h_{e2}}{h_1 - h_f} \right)
\]

\[
-\frac{W_{net}}{\dot{m}} = \left( T_1 (s_1 - s_2) - (h_1 - h_2) \right) - x_1 (h_3 - h_{e1}) - x_2 (h_5 - h_{e2})
\]
A self assessment exercise is given after this slide.

Kindlyasses yourself for this lecture.
Self Assessment

In a reversible Claude system, if $T_1$, $T_2$, $T_3$ are held constant,

1. The $y_{\text{max}}$ _______ with the decrease in $T_3$.
2. $W/m_{\text{fmin}}$ _______ with the decrease in $T_3$.
3. The overall inefficiency of compressor is ______
4. The overall inefficiency of an expander is ______
5. Kapitza and Heylandt systems are the modifications of the _____ System.
6. ______ system is widely used in helium liquefaction.
7. The regenerator/heat exchanger performs both _____ & _______.

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8. _______ system was the first one to use a turbo-expander.
9. _______ system is a high-pressure Air liquefaction system.
10. In a Heylandt system, the inlet to the expander is at _______.
11. _______ system is considered as one of the biggest milestones in Cryogenic Engineering.
12. The inversion temperature of Helium is around _______.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay
1. Decreases
2. Increases
3. $\eta_{\text{oval},c} = \eta_{\text{mech},c} \times \eta_{\text{iso},c}$
4. $\eta_{\text{oval},e} = \eta_{\text{mech},e} \times \eta_{\text{ad},e}$
5. Claude
6. Collins
7. Gas cooling/warming, Gas purification
8. Kapitza
9. Heylandt
Answers

10. Ambient
11. Collins
12. 45 K
Thank You!