CRYOGENIC ENGINEERING

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Lecture No - 19
• Synthetic membranes, Adsorption, Absorption and distillation are some of the common techniques of Gas Separation.

• Mixing of two different gases is an irreversible process because unmixing or separation requires work input.

• Ideal work requirement per mole of mixture to separate a mixture with $N$ constituents is given by

$$\frac{-W_i}{n_m} = RT_m \sum_{j=1}^{N} y_j \ln \left( \frac{1}{y_j} \right)$$

• where $y_j$ is mole fraction of $j^{th}$ component.
Outline of the Lecture

Topic: Gas Separation (contd)

• Ideal Gas Separation System
  • Tutorials
  • Parametric study
  • 3 – Gas mixtures
In general, the composition of any mixture can be specified in three different ways. They are:

- Volume percentage
- Weight percentage
- Mole Fraction

In most of the Gas separation problems, the correlations are based on mole fractions.

Hence, all percentages have to be ultimately expressed in the mole fractions.
Tutorial – 1

• Consider a mixture of Gas A and Gas B with a composition of 60% and 40% respectively by weight. Determine the mole fraction of Gas A and Gas B, if the temperature and the pressure of the mixture are 300 K and 1.013 bar respectively. Given that the molecular weight of Gas A and Gas B are 28g/mol and 32g/mol respectively.

• Calculate the mole fractions if the above percentages are given on volume basis.
## Tutorial – 1

### Given

- Working Pressure: 1 atm
- Temperature: 300 K

### Mixture Composition

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>60% A + 40% B by w/w.</td>
</tr>
<tr>
<td>II</td>
<td>60% A + 40% B by v/v.</td>
</tr>
</tbody>
</table>

### For above mixtures, Calculate

- $y_A$ and $y_B$
Tutorial – 1

- 60% A + 40% B w/w.

Let the mass of the mixture be $x$ gm. Then the mass of the Gas A in the mixture is $0.6x$ gm.

- The number of moles of Gas A are

- The number of moles of Gas B are

- Total moles in the mixture are

\[
\begin{align*}
    n_a &= \frac{0.6x}{28} \\
    n_b &= \frac{0.4x}{32} \\
    n_{tot} &= \frac{0.6x}{28} + \frac{0.4x}{32}
\end{align*}
\]
Tutorial – 1

- 60% A + 40% B w/w.

\[ y_a = \frac{n_a}{n_{tot}} = \frac{0.6x}{28} + \frac{0.4x}{32} \]

\[ y_a = 0.631 \]

\[ y_b = \frac{n_b}{n_{tot}} = \frac{0.4x}{28} + \frac{0.4x}{32} \]

\[ y_b = 0.369 \]
Tutorial – 1

- **60% A + 40% B v/v.**

Let the moles of **Gas A** be $n_a$. Using the Ideal Gas Law,

- Similarly, the moles of **Gas B** is

- The total moles is

\[
\begin{align*}
\text{Total moles} &= \frac{p(0.6V_t)}{\mathcal{R}T} + \frac{p(0.4V_t)}{\mathcal{R}T} \\
&= \frac{pV_t}{\mathcal{R}T}
\end{align*}
\]
Tutorial – 1

• 60% A + 40% B v/v.

\[
\text{Gas A: } \frac{n_a}{n_{tot}} = \frac{0.6 \frac{p(V_t)}{RT}}{0.6 \frac{p(V_t)}{RT} + 0.4 \frac{p(V_t)}{RT}}
\]

\[
y_a = 0.6
\]

\[
\text{Gas B: } \frac{n_b}{n_{tot}} = \frac{0.4 \frac{p(V_t)}{RT}}{0.6 \frac{p(V_t)}{RT} + 0.4 \frac{p(V_t)}{RT}}
\]

\[
y_b = 0.4
\]

Volume fraction = mole fraction
The Ideal work of separation per mole of mixture (Gas A and Gas B) is given by

\[ -\frac{W_{i,m}}{n_m} = R T_m \left( y_a \ln \left( \frac{1}{y_a} \right) + y_b \ln \left( \frac{1}{y_b} \right) \right) \]

On the similar lines, if the mixture is composed of three different gases, say Gas A, Gas B, and Gas C, the Ideal work of separation per mole of mixture is given by

\[ -\frac{W_{i,m}}{n_m} = R T_m \left( y_a \ln \left( \frac{1}{y_a} \right) + y_b \ln \left( \frac{1}{y_b} \right) + y_c \ln \left( \frac{1}{y_c} \right) \right) \]

Where \( y_a, y_b \) and \( y_c \) are the mole fractions of Gas A, Gas B, and Gas C respectively.
Tutorial – 2

• Determine the $W_{i,m}/n_m$, $W_{i,m}/n_{N_2}$, $W_{i,m}/n_{O_2}$ for the separation of mixture of gases consisting of 80% $N_2$ and 20% $O_2$ by mole fraction. The mixture is at 300 K and a pressure of 1.013 bar (1 atm). The mol. wt. of $N_2$ and $O_2$ are 28 and 32 g/mol respectively.

• For the above problem, also calculate the ideal work requirement for the unit mass of $N_2$ and $O_2$ respectively.
# Tutorial – 2

**Given**

- Working Pressure: 1 atm
- Temperature: 300 K
- Mixture: 80% N₂ + 20% O₂ by mole fraction

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wₘ,m/nₘ</td>
<td>Work of separation of mixture/mole of mixture</td>
</tr>
<tr>
<td>Wₘ,m/nO₂</td>
<td>Work of separation of mixture/mole of Oxygen</td>
</tr>
<tr>
<td>Wₘ,m/nN₂</td>
<td>Work of separation of mixture/mole of Nitrogen</td>
</tr>
<tr>
<td>Wₘ,m/mO₂</td>
<td>Work of separation of mixture/mass of Oxygen</td>
</tr>
<tr>
<td>Wₘ,m/mN₂</td>
<td>Work of separation of mixture/mass of Nitrogen</td>
</tr>
</tbody>
</table>
Tutorial – 2

**Ideal Work/mole of mixture**

\[
\frac{-W_{i,m}}{n_m} = R T_m \left( y_a \ln \left( \frac{y_a}{y_a} \right) + y_b \ln \left( \frac{y_b}{y_b} \right) \right)
\]

**Data**

\( R = 8.314 \text{ J/mol} - \text{ K} \)

\( T_m = 300 \text{ K} \)

\( y_a = 0.8 \text{ (mole fraction of N}_2\text{)} \)

\( y_b = 0.2 \text{ (mole fraction of O}_2\text{)} \)

\[
\frac{-W_{i,m}}{n_m} = (8.314)(300) \left( 0.8 \ln \left( \frac{1}{0.8} \right) + 0.2 \ln \left( \frac{1}{0.2} \right) \right) = 1248.1 \text{ J/mol}
\]
• **Ideal Work/mole of N₂**

\[
\frac{-W_{i,m}}{n_{N₂}} = \left( \frac{-W_{i,m}}{n_m} \right) \left( \frac{n_m}{n_{N₂}} \right) \left( \frac{n_m}{n_{N₂}} \right) = \frac{1}{y_{N₂}}
\]

\[
\frac{-W_{i,m}}{n_{N₂}} = \left( \frac{-W_{i,m}}{n_m} \right) \left( \frac{1}{y_{N₂}} \right)
\]

\[
\frac{-W_{i,m}}{n_m} = 1248.1
\]

\[
y_{N₂} = 0.8
\]

\[
\frac{-W_{i,m}}{n_{N₂}} = \frac{1248.1}{0.8} = 1560.1 \text{ } J/mol - N₂
\]
Tutorial – 2

• Ideal Work/mole of $O_2$

\[
\frac{-W_{i,m}}{n_{O2}} = \left( \frac{-W_{i,m}}{n_m} \right) \left( \frac{1}{y_{O2}} \right)
\]

\[
\frac{-W_{i,m}}{n_m} = 1248.1
\]

\[
y_{O2} = 0.2
\]

\[
\frac{-W_{i,m}}{n_{O2}} = \frac{1248.1}{0.2} = 6240.5 \text{ J/mol} - O_2
\]
Tutorial – 2

• Ideal Work/mass of N₂

\[
\left( \frac{-W_{i,m}}{m_{N₂}} \right) = \left( \frac{-W_{i,m}}{n_{N₂}} \right) \left( \frac{n_{N₂}}{m_{N₂}} \right) = \left( \frac{1}{molw_{N₂}} \right) \]

\[
\frac{-W_{i,m}}{m_{N₂}} = \left( \frac{-W_{i,m}}{n_{N₂}} \right) \left( \frac{1}{molw_{N₂}} \right)
\]

\[
\frac{-W_{i,m}}{n_{N₂}} = 1560.1
\]

\[
molw_{N₂} = 28
\]

\[
\frac{-W_{i,m}}{m_{N₂}} = \frac{1560.1}{28} = 55.71 \text{ J/gm} - N₂
\]
Tutorial – 2

• Ideal Work/mass of O₂

\[
\frac{-W_{i,m}}{m_{O₂}} = \left(\frac{-W_{i,m}}{n_{O₂}}\right)\left(\frac{1}{\text{molw}_{O₂}}\right)
\]

\[
\frac{-W_{i,m}}{n_{O₂}} = 6240.5 \quad \text{molw}_{O₂} = 32
\]

\[
\frac{-W_{i,m}}{m_{O₂}} = \frac{6240.5}{32} = 195.01 J / gm \text{ – O₂}
\]
Tabulating the results, we have

<table>
<thead>
<tr>
<th>Work</th>
<th>300 K, 1 atm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{i,m}/n_m$</td>
<td>1248.1</td>
</tr>
<tr>
<td>$W_{i,m}/n_{N_2}$</td>
<td>1560.1</td>
</tr>
<tr>
<td>$W_{i,m}/n_{O_2}$</td>
<td>6240.5</td>
</tr>
</tbody>
</table>

$W_{i,m}/n_m$ is always less than the $W_{i,m}/n_{N_2}$ or $W_{i,m}/n_{O_2}$ because $n_m = n_{N_2} + n_{O_2}$. 
As mentioned earlier, the ideal work of separation for a mixture of **Gas A** and **Gas B** is given by

\[
\frac{-W_{i,m}}{n_m} = RT_m \left( y_a \ln \left( \frac{1}{y_a} \right) + y_b \ln \left( \frac{1}{y_b} \right) \right)
\]

Since, \( y_a \) and \( y_b \) are the mole fractions of **Gas A** and **Gas B** respectively, the following condition is true at all times.

\[
y_a + y_b = 1 \quad \Rightarrow \quad y_b = 1 - y_a
\]

Substituting, we have

\[
\frac{-W_{i,m}}{n_m} = RT_m \left( y_a \ln \left( \frac{1}{y_a} \right) + (1 - y_a) \ln \left( \frac{1}{1 - y_a} \right) \right)
\]
Parametric Study

\[-\frac{W_{i,m}}{n_m} = y_a T_m \left( y_a \ln \left( \frac{1}{y_a} \right) + (1 - y_a) \ln \left( \frac{1}{1-y_a} \right) \right)\]

- It is clear that the ideal work of separation for a mixture is dependent on the mole fractions \((y_a\) and \(y_b)\) of **Gas A** and **Gas B** respectively.

- Also, the work requirement decreases with the decrease in the temperature.

- The effect of \(y_a\) and the separation temperature on the ideal work requirement is studied in a greater detail through the next tutorial.
Tutorial – 3

• Consider mixtures of **Gas A** and **Gas B** with the following compositions in mole fractions.

<table>
<thead>
<tr>
<th>Mixture Composition</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>30% A + 70% B</td>
</tr>
<tr>
<td>II</td>
<td>50% A + 50% B</td>
</tr>
<tr>
<td>III</td>
<td>60% A + 40% B</td>
</tr>
<tr>
<td>IV</td>
<td>80% A + 20% B</td>
</tr>
</tbody>
</table>

• Determine the $W_{i,m}/n_m$, $W_{i,m}/n_A$, $W_{i,m}/n_B$ for the separation of this mixture given that the mixture is at 300 K and 200 K. The mixture pressure is 1.013 bar (1 atm).
## Tutorial – 3

### Given

Working Pressure : 1 atm  
Temperature : 300 K and 200 K

### Mixture Composition

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
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<td></td>
</tr>
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</tr>
<tr>
<td>IV</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

### For above mixtures, Calculate

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{i,m}/n_m )</td>
<td>Work of separation of mixture/mole of mixture</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( W_{i,m}/n_A )</td>
<td>Work of separation of mixture/mole of Gas A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( W_{i,m}/n_B )</td>
<td>Work of separation of mixture/mole of Gas B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Methodology

• The two separation temperatures under study are 300 K and 200 K.

• In this tutorial, the $W_{i,m}/n_m$, $W_{i,m}/n_A$, $W_{i,m}/n_B$ are calculated only for 300 K and mixture III.

• All other calculations pertaining to 200 K and other mixtures are left as an exercise to the students.

• The data is plotted graphically in the further slides.
Tutorial – 3

• Ideal Work/mole of mixture – III

\[
-\frac{W_{i,m}}{n_m} = \mathcal{R}T_m \left( y_A \ln \left( \frac{1}{y_A} \right) + y_B \ln \left( \frac{1}{y_B} \right) \right)
\]

Data

\( \mathcal{R} = 8.314 \text{ J/mol} - \text{K} \)
\( T_m = 300 \text{ K} \)
\( y_A = 0.6 \) (mole fraction of A)
\( y_B = 0.4 \) (mole fraction of B)

\[
-\frac{W_{i,m}}{n_m} = (8.314)(300) \left( 0.6 \ln \left( \frac{1}{0.6} \right) + 0.4 \ln \left( \frac{1}{0.4} \right) \right) = 1678.6 \text{ J/mol}
\]
• Ideal Work/mole of A

\[
\frac{-W_{i,m}}{n_A} = \left( \frac{-W_{i,m}}{n_m} \right) \left( \frac{1}{y_A} \right)
\]

\[
\frac{-W_{i,m}}{n_m} = 1678.6
\]

\[
y_A = 0.6
\]

\[
\frac{-W_{i,m}}{n_A} = \frac{1678.6}{0.6} = 2797.6 \text{ J/ mol} - A
\]
Tutorial – 3

• Ideal Work/mole of B

\[
\frac{-W_{i,m}}{n_B} = \left( \frac{-W_{i,m}}{n_m} \right) \left( \frac{1}{y_B} \right)
\]

\[
\frac{-W_{i,m}}{n_m} = 1678.6
\]

\[y_B = 0.4\]

\[
\frac{-W_{i,m}}{n_B} = \frac{1678.6}{0.4} = 4196.5 \text{ J/mol} - B
\]
Tutorial – 3

- Extending the calculations for all other mixtures at 300 K temperature, we have the following table.

<table>
<thead>
<tr>
<th>300 K</th>
<th>$W_{i,m}/n_m$</th>
<th>$W_{i,m}/n_A$</th>
<th>$W_{i,m}/n_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3A + 0.7B</td>
<td>1523</td>
<td>5078</td>
<td>2176</td>
</tr>
<tr>
<td>0.5A + 0.5B</td>
<td>1728</td>
<td>3457</td>
<td>3457</td>
</tr>
<tr>
<td>0.6A + 0.4B</td>
<td>1678</td>
<td>2797</td>
<td>4196</td>
</tr>
<tr>
<td>0.8A + 0.2B</td>
<td>1248</td>
<td>1560</td>
<td>6240</td>
</tr>
</tbody>
</table>

- For any given mixture, applying the same analogy, the $W_{i,m}/n_m$ is always less as compared to either $W_{i,m}/n_A$ or $W_{i,m}/n_B$. 
Similarly, the calculations for all other mixtures at 200 K temperature, the results are as tabulated below.

<table>
<thead>
<tr>
<th>200 K</th>
<th>$W_{i,m}/n_m$</th>
<th>$W_{i,m}/n_A$</th>
<th>$W_{i,m}/n_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3A + 0.7B</td>
<td>1015</td>
<td>3385</td>
<td>1451</td>
</tr>
<tr>
<td>0.5A + 0.5B</td>
<td>1152</td>
<td>2305</td>
<td>2305</td>
</tr>
<tr>
<td>0.6A + 0.4B</td>
<td>1119</td>
<td>1865</td>
<td>2797</td>
</tr>
<tr>
<td>0.8A + 0.2B</td>
<td>832</td>
<td>1040</td>
<td>4160</td>
</tr>
</tbody>
</table>
• $W_{i,m}/n_m$ v/s. $y_A$

![Ideal Separation System](image)

- The Plot for $W_{i,m}/n_m$ versus $y_A$ for 300 K and 200 K are as shown.

<table>
<thead>
<tr>
<th></th>
<th>$y_A$</th>
<th>$W_{i,m}/n_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>300K</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.3</td>
<td>1523</td>
</tr>
<tr>
<td>II</td>
<td>0.5</td>
<td>1728</td>
</tr>
<tr>
<td>III</td>
<td>0.6</td>
<td>1678</td>
</tr>
<tr>
<td>IV</td>
<td>0.8</td>
<td>1248</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$y_A$</th>
<th>$W_{i,m}/n_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>200K</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.3</td>
<td>1015</td>
</tr>
<tr>
<td>II</td>
<td>0.5</td>
<td>1152</td>
</tr>
<tr>
<td>III</td>
<td>0.6</td>
<td>1119</td>
</tr>
<tr>
<td>IV</td>
<td>0.8</td>
<td>832</td>
</tr>
</tbody>
</table>
• $W_{i,m}/n_m$ v/s. $y_A$

- It is clear that the $W_{i,m}/n_m$ decreases with the decrease in the separation temperature.

- Also for a given mixture and temperature, the $W_{i,m}/n_m$ crosses a maxima.

- This maxima occurs when the mole fractions of Gas A and Gas B are equal.
In case of a three gas mixture say Gas A, B and C, the horizontal position of maxima occurs when

\[ y_A = y_B = y_C = 0.33 \]

This position of the maxima is independent of the separation temperature.
Tutorial – 3

- **$W_{i,m}/n_A$ v/s. $y_A$**

- The Plot for $W_{i,m}/n_A$ versus $y_A$ for 300 K and 200 K are as shown.

### 300K

<table>
<thead>
<tr>
<th></th>
<th>$y_A$</th>
<th>$W_{i,m}/n_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.3</td>
<td>5078</td>
</tr>
<tr>
<td>II</td>
<td>0.5</td>
<td>3457</td>
</tr>
<tr>
<td>III</td>
<td>0.6</td>
<td>2797</td>
</tr>
<tr>
<td>IV</td>
<td>0.8</td>
<td>1560</td>
</tr>
</tbody>
</table>

### 200K

<table>
<thead>
<tr>
<th></th>
<th>$y_A$</th>
<th>$W_{i,m}/n_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.3</td>
<td>3385</td>
</tr>
<tr>
<td>II</td>
<td>0.5</td>
<td>2305</td>
</tr>
<tr>
<td>III</td>
<td>0.6</td>
<td>1865</td>
</tr>
<tr>
<td>IV</td>
<td>0.8</td>
<td>1040</td>
</tr>
</tbody>
</table>
• $W_{i,m}/n_A$ v/s. $y_A$

It is clear that the $W_{i,m}/n_A$ decreases with the decrease in the separation temperature.

Also for a given mixture and temperature, there is a steep decrease in the $W_{i,m}/n_A$ with the increase in the concentration of a particular ingredient (here Gas A).
• For a mixture with two ingredients, the separation results into two separate components.

• But, for a mixture with three ingredients, say **Gas A, Gas B** and **Gas C**, the following cases of separation are possible.
  • All three gases are separated from each other.
  • Only one gas is separated leaving the other two mixed.

• The following tutorial is taken up to have a better understanding of these concepts.
Tutorial – 4

• Consider Air as a mixture of 78% N₂, 21% O₂ and 1% Argon by mole fractions. Determine the work requirement per unit mole of Argon, when all the three gases are separated and only when Argon is separated. The mixture is at 300 K and at a pressure of 1.013 bar (1 atm).

• For the above problem, calculate the above parameters for the case of Oxygen. Comment on the results.
# Tutorial – 4

## Given

<table>
<thead>
<tr>
<th>Working Pressure</th>
<th>1 atm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>300 K</td>
</tr>
<tr>
<td>Mixture</td>
<td>78% N₂ + 21% O₂ + 1% (mol. fr.)</td>
</tr>
</tbody>
</table>

## Calculate

<table>
<thead>
<tr>
<th>(W_{i,m}/n_m)</th>
<th>(W_{i,m}/n_{Ar})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_{i,Ar}/n_m)</td>
<td>(W_{i,m}/n_{O2})</td>
</tr>
<tr>
<td>(W_{i,O2}/n_m)</td>
<td>(W_{i,Ar}/n_{Ar})</td>
</tr>
<tr>
<td></td>
<td>(W_{i,O2}/n_{O2})</td>
</tr>
</tbody>
</table>
Tutorial – 4

- \( \frac{W_{i,m}}{n_m} \)

\[
\frac{-W_{i,m}}{n_m} = \mathcal{R} T_m \left( y_{Ar} \ln \left( \frac{1}{y_{Ar}} \right) + y_{O2} \ln \left( \frac{1}{y_{O2}} \right) + y_{N2} \ln \left( \frac{1}{y_{N2}} \right) \right)
\]

### Data

<table>
<thead>
<tr>
<th>( \mathcal{R} )</th>
<th>8.314 J/mol – K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_m )</td>
<td>300 K</td>
</tr>
</tbody>
</table>

### Data

<table>
<thead>
<tr>
<th>( y_{N2} )</th>
<th>0.78 ( (N_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{O2} )</td>
<td>0.21 ( (O_2) )</td>
</tr>
<tr>
<td>( y_{Ar} )</td>
<td>0.01 ( (Ar) )</td>
</tr>
</tbody>
</table>

\[
\frac{-W_{i,m}}{n_m} = (8.314)(300) \left( 0.78 \ln \left( \frac{1}{0.78} \right) + 0.21 \ln \left( \frac{1}{0.21} \right) + 0.01 \ln \left( \frac{1}{0.01} \right) \right) = 1415.6 \text{ J/mol}
\]
**Tutorial – 4**

- \( \frac{-W_{i,m}}{n_m} = 1415.6 \)

- \( y_{Ar} = 0.01 \)

\[
\frac{-W_{i,m}}{n_{Ar}} = \frac{1415.6}{0.01} = 141560 \text{ } J/mol - Ar
\]
### Tutorial – 4

- \( \frac{W_{i,m}}{n_{O_2}} \)

\[
\frac{-W_{i,m}}{n_m} = 1415.6 \quad \text{and} \quad y_{O_2} = 0.21
\]

\[
\frac{-W_{i,m}}{n_{O_2}} = \frac{1415.6}{0.21} = 6740.9 \text{ J/mol} - O_2
\]
Tutorial – 4

• \( W_{i,\text{Ar}}/n_m \)

\[
\frac{-W_{i,\text{Ar}}}{n_m} = R T_m \left( y_{\text{Ar}} \ln \left( \frac{1}{y_{\text{Ar}}} \right) + y_{\text{O}_2+N_2} \ln \left( \frac{1}{y_{\text{O}_2+N_2}} \right) \right)
\]

**Data**

\( R = 8.314 \text{ J/mol} - \text{K} \)

\( T_m = 300 \text{ K} \)

\( y_{\text{Ar}} = 0.01 \) (Ar)

\( y_{\text{O}_2+N_2} = 0.99 \) (N\textsubscript{2}+O\textsubscript{2})

\[
\frac{-W_{i,\text{Ar}}}{n_m} = (8.314)(300)\left( 0.99 \ln \left( \frac{1}{0.99} \right) + 0.01 \ln \left( \frac{1}{0.01} \right) \right) = 139.6 \text{ J/mol}
\]
Tutorial – 4

• $W_{i, Ar}/n_{Ar}$

\[
-\frac{W_{i, Ar}}{n_m} = 139.6
\]

\[
y_{Ar} = 0.01
\]

\[
-\frac{W_{i, Ar}}{n_{Ar}} = \frac{139.6}{0.01} = 13960 \text{ J/mol – Ar}
\]
**Tutorial – 4**

- $\frac{W_{i,O_2}}{n_m}$

\[
\frac{-W_{i,O_2}}{n_m} = RT_m \left( y_{O_2} \ln \left( \frac{1}{y_{O_2}} \right) + y_{Ar+N_2} \ln \left( \frac{1}{y_{Ar+N_2}} \right) \right)
\]

**Data**

- $R = 8.314 \text{ J/mol} - \text{K}$
- $T_m = 300 \text{ K}$
- $y_{O_2} = 0.21 (O_2)$
- $y_{N2+Ar} = 0.78 (N_2+Ar)$

\[
\frac{-W_{i,O_2}}{n_m} = (8.314)(300) \left( 0.78 \ln \left( \frac{1}{0.78} \right) + 0.21 \ln \left( \frac{1}{0.21} \right) \right) = 1300.8 \text{ J/mol}
\]
• \( \frac{W_{i,O_2}}{n_{O_2}} \)

\[
\frac{-W_{i,O_2}}{n_m} = 1300.8 \quad y_{O_2} = 0.21
\]

\[
\frac{-W_{i,O_2}}{n_{O_2}} = \frac{1300.8}{0.21} = 6194.3 \text{ J/mol} - O_2
\]
Tutorial – 4

- Tabulating the results, we have

<table>
<thead>
<tr>
<th>Work</th>
<th>300 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{i,m}/n_m$</td>
<td>1415.6</td>
</tr>
<tr>
<td>$W_{i,Ar}/n_m$</td>
<td>139.6</td>
</tr>
<tr>
<td>$W_{i,O_2}/n_m$</td>
<td>1300.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Work</th>
<th>300 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{i,m}/n_{Ar}$</td>
<td>141560</td>
</tr>
<tr>
<td>$W_{i,m}/n_{O_2}$</td>
<td>6740.9</td>
</tr>
<tr>
<td>$W_{i,Ar}/n_{Ar}$</td>
<td>13960</td>
</tr>
<tr>
<td>$W_{i,O_2}/n_{O_2}$</td>
<td>6194.3</td>
</tr>
</tbody>
</table>
Summary

• In general, the composition of any mixture can be specified in three different ways. They are Volume percentage, Weight percentage and Mole Fraction.

• Work/mole of mixture is always less than work/mole of its constituents for any mixture.

• \( W_{i,m}/n_m \) is maximum when the percentage compositions of all its ingredients are equal.

• With the decrease in the percentage, the work/mole of that component increases.
Assignment

• Consider a mixture of 75% \( A \), 22% \( B \) and 3% \( C \) by mole fractions. Determine the work requirement per unit mole of \( C \), when all the three gases are separated and only when \( C \) is separated. The mixture is at 300 K and at a pressure of 1.013 bar (1 atm).

• For the above problem, calculate the above parameters for the case of \( B \). Comment on the results.
## Answers

<table>
<thead>
<tr>
<th>Work</th>
<th>300 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{i,m}/n_m$</td>
<td>1631.3</td>
</tr>
<tr>
<td>$W_{i,c}/n_m$</td>
<td>336.0</td>
</tr>
<tr>
<td>$W_{i,B}/n_m$</td>
<td>1314.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Work</th>
<th>300 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{i,m}/n_C$</td>
<td>54379</td>
</tr>
<tr>
<td>$W_{i,m}/n_B$</td>
<td>2471.7</td>
</tr>
<tr>
<td>$W_{i,c}/n_C$</td>
<td>11202</td>
</tr>
<tr>
<td>$W_{i,B}/n_B$</td>
<td>5973.6</td>
</tr>
</tbody>
</table>
Thank You!