CRYOGENIC ENGINEERING

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Lecture No - 23
Earlier Lecture

• Earlier, we have studied the Temperature composition diagrams, the Enthalpy composition diagrams and their importance in Gas separation.

• The separation of a mixture is more effective when the difference in the boiling points is more.

• In this column, Low and High Boiling components are collected at top and bottom respectively.

• **Murphree efficiency** is the ratio of actual change in mole fraction to the maximum possible change that can occur.
Outline of the Lecture

Topic: Gas Separation (contd)

- Understanding of Rectification Column using an Animation
- Theoretical Plate Calculations
Rectification Column

- Animation
As seen earlier in a rectification column, the liquid moving down is enriched in high boiling point component (O₂).

On the other hand, the vapor moving up is enriched in low boiling point component (N₂).
For getting 100% pure products, infinite number of rectification processes – plates, would be required.

But in reality, the size and the cost of the column limit the number of rectification processes and hence the purity.
Rectification Column

• In the past, researchers have developed various mathematical procedures to calculate the required number of rectification processes – plates, to obtain a desired purity.

• These procedures require the following data.
  • Number of components
  • Phase diagrams of the mixtures
  • Property data of mixture
  • Heat transfer correlations
Theoretical Plate Calculations

- The methods of calculation that are used for theoretical plate calculations are
  
  - Method of Ponchon and Savarit.
  
  - Method of McCabe and Thiele.
  
  - Numerical Methods.
Theoretical Plate Calculations

- **Ponchon – Savarit** method is an exact method for plate calculations.

- It is applicable to any number of components and this method requires a detailed data of enthalpy composition diagram(s) of the mixture.
Theoretical Plate Calculations

• **McCabe – Thiele** method was proposed by two American scientists, Warren McCabe and Ernest Thiele, in the year 1925.

• This method is less general and is the simplest technique. It is widely used for binary mixtures at cryogenic temperatures.
Theoretical Plate Calculations

- **Numerical** methods are the latest techniques, which are tedious, time consuming and computer intensive methods.

- For the sake of understanding and simplicity, only **McCabe – Thiele** method will be explained in this topic.
McCabe – Thiele Method

- This method calculates liquid and vapor fractions of each component at every plate and also the number of plates.

- For the sake of understanding, let the plates above the feed be denoted by subscript $n$.

- Similarly, the plates below the feed be denoted by subscript $m$.

- Let the total mole flow rate of top and bottom product be $D$ and $B$ respectively.
McCabe – Thiele Method

- It is important to understand the indexing pattern of the plate and its corresponding liquid and vapor.

- Let \( j^{th} \) and \( (j+1)^{th} \) plate be any intermediate plate as shown in the figure.

- The **liquid** and **vapor** leaving from top of the \( j^{th} \) plate are \( L_j \) and \( V_j \) respectively.
Similarly, the liquid coming to the \( j \)th plate is from \((j+1)\)th plate, therefore it is \( L_{j+1} \).

Also, the vapor coming to \( j \)th plate from bottom is vapor leaving the \((j-1)\)th plate. It is therefore, \( V_{j-1} \).

The vapor and liquid on any plate, \( L_j \) and \( V_j \), are in thermal equilibrium.
McCabe – Thiele Method

- Consider a control volume enclosing the condenser and the top section of the \( n^{th} \) plate as shown in the figure.

- As explained earlier, for this \( n^{th} \) plate, the vapor leaving is \( V_n \) and the liquid added is \( L_{n+1} \).

- Applying the mole balance across the control volume per unit time, we have

\[
\begin{array}{c|c}
\text{IN} & \text{OUT} \\
V_n & L_{n+1} \\
D & D
\end{array}
\]

\[V_n = L_{n+1} + D\]
McCabe – Thiele Method

• Multiplying the mole balance equation with mole fraction of a particular component in a mixture, we get mole balance for that component as

\[ y_n V_n = x_{n+1} L_{n+1} + x_D D \]

• Where,
  • \( y_n \), \( x_{n+1} \) and \( x_D \) are mole fractions of a particular component in vapor, liquid and top product respectively.

• It automatically means that \( x_D \) (mole fraction) is the desired purity of the top product.
McCabe – Thiele Method

- For control volume taking into account $Q_D$ (watts) as the heat rejected by the condenser, the enthalpy balance is given by:

$$H_n V_n = h_{n+1} L_{n+1} + h_D D + \dot{Q}_D$$

- Dividing the above equation by $D$, we have:

$$\frac{H_n V_n}{D} = h_{n+1} \frac{L_{n+1}}{D} + h_D + \frac{\dot{Q}_D}{D}$$

- Rearranging the total mole balance equation, we have:

$$L_{n+1} = V_n - D$$

$$\frac{L_{n+1}}{D} = \frac{V_n}{D} - 1$$
McCabe – Thiele Method

- Eliminating $L_{n+1}/D$ from the earlier equations, we get

$$\frac{H_n V_n}{D} = h_{n+1} \left( \frac{V_n}{D} - 1 \right) + h_D + \frac{\dot{Q}_D}{D}$$

$$\left( H_n - h_{n+1} \right) \frac{V_n}{D} = \frac{\dot{Q}_D}{D} + h_D - h_{n+1}$$

- Rearranging as a ratio of $D$ and $V_n$, we have

$$\frac{D}{V_n} = \frac{H_n - h_{n+1}}{\dot{Q}_D/D + h_D - h_{n+1}}$$
McCabe – Thiele Method

The enthalpy composition diagram for a mixture of \( \text{N}_2 \) and \( \text{O}_2 \) is as shown.

If we neglect the enthalpy variation with the mole fraction, the bubble and dew lines can be taken as horizontal.
McCabe – Thiele Method

\[ \frac{D}{V_n} = \frac{H_n - h_{n+1}}{\frac{\dot{Q}_D}{D} + h_D - h_{n+1}} \]

\[ \frac{L_{n+1}}{V_n} = 1 - \frac{D}{V_n} \]

• These arguments lead to the fact that liquid \((h)\) and vapor \((H)\) enthalpies are constant. Hence, \(D/V_n\) and \(L_{n+1}/V_n\) are constant.

• Rearranging the molar balance for a component as

\[ y_n = \left(\frac{L_{n+1}}{V_n}\right)x_{n+1} + \left(\frac{D}{V_n}\right)x_D \]

• The above equation represents a straight line and is called as Operating Line for stripping section.
McCabe – Thiele Method

\[ y_n = \left( \frac{L_{n+1}}{V_n} \right) x_{n+1} + \left( \frac{D}{V_n} \right) x_D \]

- For the top or upper most plate near the condenser, \( x_{n+1} = x_D \).
- Substituting,

\[ y_n = \left( \frac{L_{n+1}}{V_n} \right) x_D + \left( \frac{D}{V_n} \right) x_D \]

- For y – intercept, \( x_{n+1} = 0 \).

\[ y_n = \left( \frac{D}{V_n} \right) x_D \]

Two Points

- \( y_n = x_D @ x_{n+1} = x_D \)
- \( y_n = (D/V_n)x_D @ x_{n+1} = 0 \)
McCabe – Thiele Method

• A plot of vapor versus liquid mole fractions for a particular component, say A, is as shown in the figure.

• Let 45° diagonal or \( y = x \) line be as shown.

• The desired purity of this component A, in the top product is \( x_D \) as shown in the figure.
The y – intercept of the straight line is \((D/V_n)xD\).

Similarly, the slope of the operating line is given by \(L_{n+1}/V_n\), as shown in the above equation.
• Similarly, for the analysis of $m^{th}$ plate and boiler in the lower part, we have the following equations.

  • Mole Balance: $L_{m+1} = V_m + B$

  \[ x_{m+1}L_{m+1} = y_mV_m + x_BB \]

  • Energy Balance: $h_{m+1}L_{m+1} + \dot{Q}_B = H_mV_m + h_BB$

  • where, $B$ and $Q_B$ are mole flow rate out at the bottom and heat input to the boiler respectively.
McCabe – Thiele Method

• Rearranging the above equations, we have the following.

\[ \frac{B}{V_m} = \frac{H_m - h_{m+1}}{\bar{Q}_m - h_B + h_{m+1}} \]

\[ \frac{L_{m+1}}{V_m} = 1 + \frac{B}{V_m} \]

• Applying the assumption, we have \( H_m \) and \( h_{m+1} \) as constant, implies \( B/V_m \) and \( L_{m+1}/V_m \) are constant. The operating line for stripping section is

\[ y_m = \left( \frac{L_{m+1}}{V_m} \right) x_{m+1} - \left( \frac{B}{V_m} \right) x_B \]
McCabe – Thiele Method

\[ y_m = \left( \frac{L_{m+1}}{V_m} \right) x_{m+1} - \left( \frac{B}{V_m} \right) x_B \]

- For the bottom or lower most plate near the boiler, \( x_{m+1} = x_B \).

- Substituting,

\[ y_m = \left( \frac{L_{m+1}}{V_m} \right) x_B - \left( \frac{B}{V_m} \right) x_B \]

- For \( y \) – intercept, \( x_{m+1} = 0 \).

\[ y_m = -\left( \frac{B}{V_m} \right) x_B \]

**Two Points**

\[
\begin{align*}
y_m &= x_B \quad @ \quad x_{m+1} = x_B \\
y_m &= -(B/V_m)x_B \quad @ \quad x_{m+1} = 0
\end{align*}
\]
McCabe – Thiele Method

- The plot of vapor versus liquid mole fractions for a component $A$ with operating line and $45^\circ$ diagonal be as shown.

- The purity of component $A$ in the bottom product is $x_B$ as shown in the figure.
McCabe – Thiele Method

- The y-intercept of the straight line is \(-\left(\frac{B}{V_m}\right)x_B\).

- The slope of the operating line is given by \(\frac{L_{m+1}}{V_m}\) as shown in the above equation.

\[
y_m = \frac{L_{m+1}}{V_m} x_{m+1} - \left(\frac{B}{V_m}\right)x_B
\]
The mixture that is to be separated is called as Feed. It is introduced into the column through an opening called as Feed inlet as shown in the figure.

Consider a control volume enclosing the $n^{th}$ and $m^{th}$ plates and feed inlet as shown.

Let $F$ be the total number of moles in the Feed.
McCabe – Thiele Method

• For the above control volume, we have

\[
F = V_n - V_m + L_{m+1} - L_{n+1}
\]

- Applying the molar balance

<table>
<thead>
<tr>
<th>IN</th>
<th>OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td></td>
</tr>
<tr>
<td>(V_m)</td>
<td>(V_n)</td>
</tr>
<tr>
<td>(L_{n+1})</td>
<td>(L_{m+1})</td>
</tr>
</tbody>
</table>
McCabe – Thiele Method

- We define a parameter $q$ as the ratio of liquid moles in the feed to the total number moles in the feed.

- Mathematically,

$$q = \frac{(L_{m+1} - L_{n+1})}{F}$$

- That is for $q=0$, feed is totally vapor and for $q=1$, it is totally liquid.
McCabe – Thiele Method

- From the earlier slides, we know the equations for both the sections.

- The locus of intersection of these operating lines denotes the feed condition.

- The condition of the feed is vital to determine the number of plates.
McCabe – Thiele Method

- Based on feed equation and $q$ definition, we have

$$F = V_n - V_m + L_{m+1} - L_{n+1}$$

$$q = \frac{(L_{m+1} - L_{n+1})}{F}$$

$$V_n - V_m = (1 - q)F$$

- Again, from the operating lines of upper and lower sections, we can rearrange to give $V_n$ and $V_m$ as

$$V_n = \left(\frac{L_{n+1}}{y_n}\right)x_{n+1} + \left(\frac{D}{y_n}\right)x_D$$

$$V_m = \left(\frac{L_{m+1}}{y_m}\right)x_{m+1} - \left(\frac{B}{y_m}\right)x_B$$

- It is important to note that $V_n - V_m$ is the vapor content in the feed.
In the calculation of point of intersection of operating lines, we choose a common point to both these lines as \((x, y)\).

Hence, \(x_{n+1}, x_{m+1}, y_m\) and \(y_n\) are replaced with this point as shown in the following equation.

\[
V_n - V_m = \frac{(L_{n+1} - L_{m+1})x}{y} + \frac{(x_D D + x_B B)}{y} = (1 - q)F
\]

The locus of this point of intersection is the feed line or \(q\) line and is calculated as explained in the next slide.
McCabe – Thiele Method

- For a column as a whole, using the mass balance, we can write

\[ x_F F = x_D D + x_B B \]

\[ q = \frac{(L_{m+1} - L_{n+1})}{F} \]

- Rearranging the following equations, we have

\[ \frac{(L_{n+1} - L_{m+1})}{y} x + \frac{(x_D D + x_B B)}{y} = (1 - q) F \]

\[ -qF \frac{x}{y} + \frac{x_F F}{y} = (1 - q) F \]
McCabe – Thiele Method

• Rearranging,

\[ y = \left( \frac{q}{q-1} \right)x + \frac{x_F}{1-q} \]

• The above equation represents a straight line with \( \frac{q}{q-1} \) and \( \frac{x_F}{1-q} \) as slope and \( y \) – intercept respectively.

• More importantly, it is the locus of point of intersection of operating lines. This line is called as Feed line or \( q \) line.
• It is clear that the value of parameter $q$ is yet to be determined.

• Applying energy balance to the control volume as shown in figure, we have

$$\dot{Q}_D = \dot{Q}_B,$$

$$h_F F = V_n H_n - V_m H_m + L_{m+1} h_{m+1} - L_{n+1} h_{n+1}$$

• Mathematically, McCabe – Thiele assumption is

$$H_n = H_m = H, h_{m+1} = h_{n+1} = h$$
McCabe – Thiele Method

• Upon substitution, we have

\[ h_F F = V_n H_n - V_m H_m + L_{m+1} h_{m+1} - L_{n+1} h_{n+1} \]

\[ h_F F = (V_n - V_m) H + (L_{m+1} - L_{n+1}) h \]

• Also, we have the following equations.

\[ V_n - V_m = (1 - q) F \]

\[ q = \frac{(L_{m+1} - L_{n+1})}{F} \]

• Combining the above equations and rearranging, we have

\[ q = \frac{H - h_F}{H - h} \]
McCabe – Thiele Method

Depending on the feed condition, \( q \) can take any value.

<table>
<thead>
<tr>
<th>Condition</th>
<th>( q )</th>
<th>Slp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sat. Vap. (( h_F = H ))</td>
<td>( q=0 )</td>
<td>0</td>
</tr>
<tr>
<td>Sat. Liq. (( h_F = h ))</td>
<td>( q=1 )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>2 ph. (( H &lt; h_F &lt; h ))</td>
<td>( 0 &lt; q &lt; 1 )</td>
<td>-ve</td>
</tr>
<tr>
<td>Sub. Liq. (( h_F &lt; h ))</td>
<td>( q &gt; 1 )</td>
<td>+ve</td>
</tr>
<tr>
<td>Sup. Vap. (( h_F &gt; h ))</td>
<td>( q &lt; 0 )</td>
<td>+ve</td>
</tr>
</tbody>
</table>
The equation of feed line is

\[ y = \left( \frac{q}{q-1} \right) x + \frac{x_F}{1-q} \]

<table>
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<tr>
<th>Condition</th>
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<th>Slope (Slp)</th>
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</table>
McCabe – Thiele Method

• The point of intersection of feed line or \( q \) line and \( y=x \) gives the content of the component \( A \) in feed, \( x_F \).

• It is calculated by substituting \( y=x \) in the feed line as shown.

\[
x = \left( \frac{q}{q-1} \right) x + \frac{x_F}{1-q}
\]

\[
x = x_F
\]
McCabe – Thiele Method

- Graphically, it is easier to draw a line through two given points rather than using a given slope and a point.

- This intersection point is used to draw the feed line as shown in the figure.
Summary

• Plate calculation procedures require the data like number of components, phase diagrams, property data of the mixtures, heat transfer correlations.

• **McCabe – Thiele** method is less general and is widely used for binary mixtures at cryogenic temperatures.

• The major assumption in this method is that the liquid and vapor enthalpies are independent of mole fraction.
Summary

- The equations of operating lines for striping and enriching sections are:

\[
y_n = \left(\frac{L_{n+1}}{V_n}\right)x_{n+1} + \left(\frac{D}{V_n}\right)x_D
\]

\[
y_m = \left(\frac{L_{m+1}}{V_m}\right)x_{m+1} - \left(\frac{B}{V_m}\right)x_B
\]

- The locus of intersection of these operating lines denotes the feed condition. It is given as:

\[
y = \left(\frac{q}{q-1}\right)x + \frac{x_F}{1-q}
\]

- The point of intersection of feed line or \(q\) line and \(y=x\) gives the content of a component in the feed, \(x_F\).
Summary

- The slope of $q$ line is given by

\[
\frac{q}{q-1}
\]

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</table>
• A self assessment exercise is given after this slide.

• Kindly asses yourself for this lecture.
Self Assessment

1. McCabe – Thiele method calculates _______ & _______ of each component at every plate.
2. For a $j^{th}$ plate, the liquid and vapor leaving from top are denoted by _____ and _____ respectively.
3. The vapor and liquid on any plate are assumed to be in _______ equilibrium.
4. In McCabe – Thiele method, liquid and vapor enthalpies are assumed to be ________.
5. The slope of operating line for stripping section is given by ________.
6. The y – intercept of operating line for enriching section is given by ________.
7. Mixture that is to be separated is called as _____.

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8. q=0 when the feed is totally ______.
9. _______ and _______ are the slope and the y – intercept of q line respectively.
10. Fill the following table.

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</tr>
</tbody>
</table>
1. Vapor fraction, liquid fraction
2. $L_j$ and $V_j$
3. Thermal
4. Constant
5. $L_{n+1}/V_n$
6. $-(B/V_m)x_B$
7. Feed
8. Vapor
9. $q/(q-1)$ and $x_F/(1-q)$

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Thank You!