Earlier Lecture

• In the earlier lecture, we have seen the working of rectification column with the help of animation.

• **Ponchon & Savarit, McCabe & Thiele** and **Numerical** techniques are used to calculate the theoretical number of plates.

• **McCabe & Thiele** method is less general and is widely used for binary mixtures.

• The major assumption is that the saturated vapor and saturated liquid enthalpies are independent of the mole fraction.
In the earlier lecture, the equations of operating lines for striping and enriching sections are derived.

The locus of intersection of these operating lines denotes the feed condition.

The point of intersection of feed line or $q$ line and $y=x$ gives the content of a component in the feed, $x_F$. 
Outline of the Lecture

Topic : Gas Separation (contd)

- Graphical solution for column design using McCabe – Thiele method
- Tutorial
McCabe – Thiele Method

- Plot of vapor and liquid mole fractions for a particular component for OP line is shown.

- Let $45^\circ$ diagonal or $y=x$ line be as shown.

- The desired purity and impurity of this component in top and bottom products are $x_D$ and $x_B$ respectively.
McCabe – Thiele Method

- The equation of operating line for stripping section is

\[ y_m = \left( \frac{L_{m+1}}{V_m} \right) x_{m+1} - \left( \frac{B}{V_m} \right) x_B \]

- The equation for operating line in enriching section is

\[ y_n = \left( \frac{L_{n+1}}{V_n} \right) x_{n+1} + \left( \frac{D}{V_n} \right) x_D \]
McCabe – Thiele Method

- Let this point of intersection be \( O \).

- The feed line equation is

\[
y = \left( \frac{q}{q-1} \right)x + \frac{x_F}{1-q}
\]

<table>
<thead>
<tr>
<th>Condition</th>
<th>( q )</th>
<th>Slp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sat. Vap. ((h_F=H))</td>
<td>( q=0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>Sat. Liq. ((h_F=h))</td>
<td>( q=1 )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>2 ph. ((H&lt;h_F&lt;h))</td>
<td>( 0&lt;q&lt;1 )</td>
<td>-ve</td>
</tr>
<tr>
<td>Sub. Liq. ((h_F&lt;h))</td>
<td>( q&gt;1 )</td>
<td>+ve</td>
</tr>
<tr>
<td>Sup. Vap. ((h_F&gt;h))</td>
<td>( q&lt;0 )</td>
<td>+ve</td>
</tr>
</tbody>
</table>
McCabe – Thiele Method

- The point of intersection of feed line or q line and y=x, gives the content of the component A in feed, $x_F$.

- This intersection point is used to draw the feed line as shown in the figure.
McCabe – Thiele Method

- The variation of equilibrium vapor and liquid fractions of a particular component (here, $A$), is called as equilibrium curve.

- It means that at any point on this curve, the vapor and liquid of this component are at same temperature.
McCabe – Thiele Method

- On each plate, vapor and liquid are in thermal equilibrium. Therefore, the plate condition lies on the equilibrium line.

- That is, equilibrium curve gives the relation between liquid composition \((x_n)\) and vapor composition \((y_n)\) on the same plate.
Since, the top and the bottom products have different boiling points, there is a gradual variation of temperature across the length of the column.

The OP lines relate the variation of liquid and vapor mole fractions of a particular component across the length of the column.
McCabe – Thiele Method

- Reviewing the OP line equation, say for the top section, it is clear that this equation relates $x_{n+1}$ and $y_n$, for this component, $A$. 

\[ y_n = \left( \frac{L_{n+1}}{V_n} \right) x_{n+1} + \left( \frac{D}{V_n} \right) x_D \]
McCabe – Thiele Method

- All these curves and lines are vital in calculating the number of plates using McCabe – Thiele method.

- In view of this, the slopes of operating and \( q \) lines, equilibrium curve and purity requirement form the basis to determine the number of plates.
The plate calculation method involves a stair casing method as explained below.

The condensate \( D \) collected at the top, has a mole fraction of component \( A \) as \( x_D \).
The liquid coming onto the top plate from the condenser has a mole fraction of $x_D$. Hence, the point $x_D$ lies on the OP line as shown in the figure.
For this plate, liquid and vapor are in thermal equilibrium. Therefore, the corresponding liquid fraction $x_n$, lies on the equilibrium curve.
This equilibrium point is found by extending a horizontal line from $x_D$ to the equilibrium curve. Let this point be denoted by $P$. 
The liquid from this plate, flows over the weir and exchanges heat with the vapor coming from the top of the lower plate.
We know that, an OP line relates $x_{n+1}$ with $y_n$. Hence for this plate, the corresponding vapor fraction lies on the OP line.
• The vapor composition, $y_n$, for this plate is found by extending a vertical line from $P$ onto the OP line. Let this point be denoted by $Q$. 
McCabe – Thiele Method

- The stair casing should be continued till the stairs cross the point \( O \) (the intersection of OP lines) as shown in the figure.
McCabe – Thiele Method

• It is clear that
  • Every horizontal line gives the condition of liquid – vapor on the same plate which are in thermal equilibrium.
  • Every vertical line gives the vapor condition for the plate below the earlier plate.
McCabe – Thiele Method

- It means that, every vertical line indicates the need for the plate in the enriching section, till the stair casing crosses the point $O$.

- The same exercise could be done for the lower section, with $x_D$ as the desired impurity of the component $A$ in the bottom product.
As mentioned earlier, each vertical line indicating a plate, the total number of vertical lines in top and bottom section, together with boiler and condenser surfaces gives the total number of theoretical plates required.
McCabe – Thiele Method

- From the adjacent hypothetical figure, the total number of vertical lines are 4.

- Hence, the total number of theoretical plates can be tabulated as shown below.

<table>
<thead>
<tr>
<th>McCabe – Thiele Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
</tr>
<tr>
<td>Bottom</td>
</tr>
</tbody>
</table>
McCabe – Thiele Method

- From the adjacent figure, it is important to note that during the heat exchange process, that is along the vertical line, the liquid composition $x$ is constant.

- Similarly, along the horizontal line, the vapor composition $y$ remains constant.
McCabe – Thiele Method

- Also, note that in moving from top to bottom, the McCabe – Thiele diagram starts with an horizontal line and ends with a vertical line.

- This is because the liquid flows downwards and is represented by a vertical line.
• In the earlier lecture, we have balanced moles and enthalpy for top, bottom and mid-section separately.

• But considering the column as a whole, the following equation hold true for mole balance.

\[ F = B + D \]

• Multiplying the above equation with the mole fraction of a particular component, we get mole balance for that component.

\[ x_F F = x_B B + x_D D \]
McCabe – Thiele Method

- Similarly, for the entire column, the enthalpy or the energy balance can be written as

\[
\dot{Q}_B + h_F F = \dot{Q}_D + h_D D + h_B B
\]

- All the arguments regarding the plate calculations would be clear in tutorial as explained in next slide.
Tutorial

Consider a rectification column for $\text{N}_2$ and $\text{O}_2$ separation operating at 1 atm. Determine the number of theoretical plates required to yield 97% $\text{N}_2$ at top and 95% $\text{O}_2$ at bottom. Feed stream is 50% $\text{N}_2$ and 50% $\text{O}_2$. Molar fraction of liquid in feed stream is 0.7 mole liquid/mole mixture. The desired flow rate at the bottom product is 20 mole/sec and the heat removed in the condenser at top of the column is 500 kW.
## Tutorial

### Given

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working Pressure</td>
<td>1 atm</td>
</tr>
<tr>
<td>Mixture</td>
<td>$N_2 + O_2$</td>
</tr>
<tr>
<td>Feed stream</td>
<td>50% $N_2 + 50% O_2$</td>
</tr>
<tr>
<td>Bottom flow rate</td>
<td>20 mole/sec = $B$</td>
</tr>
<tr>
<td>Feed liq.</td>
<td>0.7 = $q$</td>
</tr>
</tbody>
</table>

### For above mixture

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reqt. of $N_2$ (top)</td>
<td>97% = $x_D$</td>
</tr>
<tr>
<td>Reqt. of $O_2$ (bottom)</td>
<td>95% = $x_B = 0.05$</td>
</tr>
<tr>
<td>Total number of theoretical plates</td>
<td></td>
</tr>
</tbody>
</table>
**Tutorial**

\[ F = B + D \]

\[ x_F F = x_B B + x_D D \]

**Mole balance**

**Mole balance for \( \text{N}_2 \)**

### Data

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_F )</td>
<td>0.5</td>
</tr>
<tr>
<td>( x_B )</td>
<td>0.05</td>
</tr>
<tr>
<td>( x_D )</td>
<td>0.97</td>
</tr>
<tr>
<td>( B )</td>
<td>20</td>
</tr>
</tbody>
</table>

\[ F = 20 + D \]

\[ 0.5F = (0.05)(20) + (0.97)D \]

**Solving, we have**

- \( F = 39.14 \) mole/sec, \( D = 19.14 \) mole/sec.
Tutorial

- Enthalpy balance

\[ \dot{Q}_B = \dot{Q}_D + h_D D + h_B B - h_F F \]

- Fraction of stream in feed

\[ q = \frac{H - h_F}{H - h} \]

- Rearranging, we have

\[ h_F = qh + (1 - q) H \]

- For 50% \( N_2 \) + 50% \( O_2 \)
  - \( h=1084 \text{ J/mol}, \ H=6992 \text{ J/mol} \)
\[ h_F = qh + (1 - q)H \]

**Data**

<table>
<thead>
<tr>
<th>h</th>
<th>1084 J/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>6992 J/m</td>
</tr>
<tr>
<td>q</td>
<td>0.7</td>
</tr>
</tbody>
</table>

\[ h_F = (0.7)1084 + (1 - 0.7)6992 \]

\[ h_F = 2856.4 J / mol \]
\[ \dot{Q}_B = \dot{Q}_D + h_D D + h_B B - h_F F \]

**Data**

<table>
<thead>
<tr>
<th>Q_D</th>
<th>500 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>h_D</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>h_B</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>h_F</td>
<td>2856.4 J/m</td>
</tr>
</tbody>
</table>

**Data**

<table>
<thead>
<tr>
<th>F</th>
<th>39.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>20</td>
</tr>
<tr>
<td>D</td>
<td>19.14</td>
</tr>
</tbody>
</table>

\[ \dot{Q}_B = (500)10^3 + (1084)(19.14) + (1084)(20) - (2856.4)(39.14) \]

\[ \dot{Q}_B = 430.6 kW \]
Tutorial

- Operating line for **Enriching Section**

\[
\frac{D}{V_n} = \frac{H_n - h_{n+1}}{\frac{\dot{Q}_D}{D} + h_D - h_{n+1}}
\]

**Data**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_D)</td>
<td>500kW</td>
</tr>
<tr>
<td>(H_n)</td>
<td>6992 J/m</td>
</tr>
<tr>
<td>(h_{n+1})</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>(h_D)</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>(D)</td>
<td>19.14 mol/s</td>
</tr>
</tbody>
</table>

\[
\frac{D}{V_n} = \frac{6992 - 1084}{500000 + 1084 - 1084} = 0.226
\]

\[
\frac{L_{n+1}}{V_n} = 1 - \frac{D}{V_n} = 1 - 0.226 = 0.773
\]
Operating line for Enriching Section

\[ y_n = 0.773 x_{n+1} + 0.226 \]

\[ y_n = \left( \frac{L}{V} \right)_{x_{n+1}} + \left( \frac{D}{V} \right)_{x_D} \]

Data

\[ X_D = 0.97 \]

\[ \frac{L_{n+1}}{V} = 0.773 \]
**Tutorial**

- Operating line for **Stripping Section**

\[
\frac{B}{V_m} = \frac{H_m - h_{m+1}}{\frac{Q_B}{B} - h_B + h_{m+1}}
\]

<table>
<thead>
<tr>
<th>Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_B)</td>
<td>430.6 kW</td>
</tr>
<tr>
<td>(H_m)</td>
<td>6992 J/m</td>
</tr>
<tr>
<td>(h_{m+1})</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>(h_B)</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>(B)</td>
<td>20 mol/s</td>
</tr>
</tbody>
</table>

\[
\frac{B}{V_m} = \frac{6992 - 1084}{(430.6)10^3 - 1084 + 1084} = 0.274
\]

\[
\frac{L_{m+1}}{V_m} = 1 + \frac{B}{V_m} = 1 + 0.274 = 1.274
\]
Tutorial

• Operating line for **Stripping Section**

\[
y_m = \left( \frac{L_{m+1}}{V_m} \right) x_{m+1} - \left( \frac{B}{V_m} \right) x_B
\]

\[
\frac{B}{V_m} = 0.274
\]

\[
\frac{L_{m+1}}{V_m} = 1.274
\]

\[
y_m = (1.274) x_{m+1} - (0.274)(0.05)
\]

\[
y_m = 1.274x_{m+1} - 0.013
\]

**Data**

| \(x_B\) | 0.05 |
• Equation of **Feed Line**

\[ q = \frac{H - h_f}{H - h} \]

\[ y = \frac{q}{q-1} x + \frac{x_F}{1-q} \]

\[ y = \frac{0.7}{0.7-1} x + \frac{0.5}{1-0.7} \]

\[ y = -2.34x + 1.67 \]

<table>
<thead>
<tr>
<th>Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>6992 J/m</td>
</tr>
<tr>
<td>( h_f )</td>
<td>2856.4 J/m</td>
</tr>
<tr>
<td>h</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>( x_F )</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Tutorial

- Summarizing, we have the following.

- OP line for enriching section:
  \[ y_n = 0.773x_{n+1} + 0.22 \]

- OP line for stripping section:
  \[ y_m = 1.274x_{m+1} - 0.013 \]

- q line:
  \[ y = -2.34x + 1.67 \]

- The stair casing procedure is shown on an excel sheet to have a better understanding of the method.
Tutorial

• From the excel sheet, it is clear that the total number of vertical lines are 9.

• Therefore, the total number of theoretical plates for this column can be tabulated as shown below.

<table>
<thead>
<tr>
<th>McCabe – Thiele Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enriching Section</td>
</tr>
<tr>
<td>3 + 1 (Condenser)</td>
</tr>
<tr>
<td>Stripping Section</td>
</tr>
<tr>
<td>6 + 1 (Boiler)</td>
</tr>
</tbody>
</table>
Summary

• Equilibrium curve gives the relation between liquid composition \((x_n)\) and vapor composition \((y_n)\) on the same plate.

• The OP lines relate the variation of liquid \((x_{n+1})\) and vapor \((y_n)\) mole fractions of a particular component across the length of the column.

• The plate calculation is a stair casing method which involves locating equilibrium conditions on equilibrium line and OP line.
Summary

• In a McCabe – Thiele diagram
  • Each horizontal line gives the condition of liquid – vapor on the same plate which are in thermal equilibrium.

  • Each vertical line gives the vapor condition for the plate with respect to liquid that leaves the earlier plate on the top.

• The total number of vertical lines in top and bottom section, together with boiler and condenser surfaces is the total number of theoretical plates required.
Thank You!