Earlier Lecture

• Cryogenic vessels use insulation to minimize all modes of heat transfer.

• Apparent thermal conductivity ($k_A$) is calculated based on all possible modes of heat transfer.

• Expanded foam is a low density, cellular structure. A gas filled powder or a fibrous insulation reduces the gas convection due to the small size of voids.

• Radiation heat transfer is reduced by using radiation shields.
Outline of the Lecture

Topic: Cryogenic Insulation (contd)

• Vacuum
• Evacuated Powders
• Opacified Powders
• Tutorial
Types of Insulation

• Expanded Foam – Mass

• Gas Filled Powders & Fibrous Materials – Mass

• Vacuum alone – Vacuum

• Evacuated Powders – Mass + Vacuum

• Opacified Powders – Mass + Vacuum + Reflective

• Multilayer Insulation – Vacuum + Reflective
Introduction

• As seen earlier, the different modes of heat transfer are Conduction, Convection and Radiation.

• If the physical matter between the hot and the cold surfaces is removed, that is, by maintaining a perfect vacuum, Conduction and Convection are eliminated.

• However, Radiation heat transfer does not require any medium and in such cases, it is the only mode of heat transfer.
Vacuum

- It is important to note that even in vacuum, there is some residual gas.

- These gas molecules contribute to the heat transfer by gaseous conduction.

- As the vacuum improves, this gas conduction decreases.

- In an ordinary conduction, a linear temperature gradient is built up. The molecules exchange heat with each other and as well as with the surfaces.
Vacuum

- But in vacuum, the mean free path ($\lambda$) of the molecules is more than the distance between the surfaces; the molecules rarely collide with each other.

- The energy is exchanged only between the surface and the colliding molecules.

- This type of heat transfer is called as free molecular conduction or residual gas conduction.

- This exists only at very low pressures or at very good vacuum.
Vacuum

- For the sake of understanding, consider two plates with temperatures $T_1$ and $T_2$, ($T_2 > T_1$) as shown.

- The gas pressure is very low in order to ensure that the mean free path ($\lambda$) of the molecules is greater than $L$.

- In such situations, the gas molecules collide only with the surfaces and exchange energy.
Consider a molecule colliding with bottom plate and leaving towards upper plate.

The gas molecule collides with this surface at $T_1$ and it transfers some energy to the surface.

It leaves the cold surface with a kinetic energy corresponding to a temperature $T'_1$, higher than $T_1$. 
Vacuum

- Again, consider a molecule colliding with upper plate and leaving towards bottom plate.

- This gas molecule collides with surface at $T_2$ and leaves at a temperature $T'_2$, lower than $T_2$.

- It is clear that, in both these impacts, thermal equilibrium is not attained. This process is repeated and contributes to free molecular conduction.
Vacuum

• In order to measure the degree of thermal equilibrium between the molecule and the surface, we define Accommodation Coefficient ($a$).

• It is a ratio of actual energy transfer to the maximum possible energy transfer.

$$ a = \frac{\text{Actual Heat Transfer}}{\text{Max Heat Transfer}} $$

• Mathematically,

• Its value depends on the gas – surface interaction and the temperature of the surface.
Vacuum

- From the figure, for the cold surface, the actual temperature change is $(T'_2 - T'_1)$.

- But, the maximum possible temperature change is $(T'_2 - T_1)$.

- By definition, the accommodation coefficient for cold plate is

$$a_1 = \frac{T'_2 - T'_1}{T'_2 - T_1}$$
Similarly, for the hot surface, the actual temperature change is $(T'_2 - T'_1)$.

But, the maximum possible temperature change is $(T_2 - T'_1)$.

Therefore, the accommodation coefficient for the hot surface is given by

$$a_2 = \frac{T'_2 - T'_1}{T_2 - T'_1}$$
Vacuum

• From the earlier slides, the accommodation coefficients are

\[ a_1 = \frac{T_2' - T_1'}{T_2' - T_1} \quad a_2 = \frac{T_2' - T_1'}{T_2 - T_1'} \]

• Rearranging the above equations, we have

\[ T_1 = T_2' - \frac{T_2' - T_1'}{a_1} \quad T_2 = \frac{T_2' - T_1'}{a_2} + T_1' \]

\[ T_2 - T_1 = \left( T_2' - T_1' \right) \left( \frac{1}{a_1} + \frac{1}{a_2} - 1 \right) \]
Similar to an emissivity factor, we define a term accommodation factor $F_a$, which is given by

$$F_a = \frac{T_2' - T_1'}{T_2 - T_1}$$

$$F_a = \frac{1}{\left(\frac{1}{a_1} + \frac{1}{a_2} - 1\right)}$$

$$T_2 - T_1 = \left(T_2' - T_1'\right)\left(\frac{1}{a_1} + \frac{1}{a_2} - 1\right)$$
Vacuum

• The approximate accommodation coefficients for concentric sphere and concentric cylinder geometries are as tabulated below.

<table>
<thead>
<tr>
<th>Temp (K)</th>
<th>He</th>
<th>H₂</th>
<th>Ne</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.29</td>
<td>0.29</td>
<td>0.66</td>
<td>0.8-0.9</td>
</tr>
<tr>
<td>78</td>
<td>0.42</td>
<td>0.53</td>
<td>0.83</td>
<td>1.0</td>
</tr>
<tr>
<td>20</td>
<td>0.59</td>
<td>0.97</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

• The subscript 1 denotes the enclosed surface and subscript 2 denotes the enclosure.
• At a given temperature, the accommodation coefficient increases with the increase in the molecular weight of the gas.

• For a given gas, the accommodation coefficient increases with the decrease in the temperature, due to better heat transfer at lower temperatures.
• From the kinetic theory of gases, the total energy of a molecule is the sum of internal energy and kinetic energy.

\[ e = U + KE \]

\[ e = \left( c_v + \frac{R}{2} \right) T \]

\[ \Delta e = \left( c_v + \frac{R}{2} \right) \Delta T \]

• Mathematically,

• where,

• \( R \) – Specific gas constant

• \( c_v = \frac{R}{(\gamma - 1)} \) – Specific heat of gas

• \( \Delta T = (T'_2 - T'_{1}) \) – Change in temperature
Vacuum

\[ \Delta e = \left( c_v + \frac{R}{2} \right) \Delta T \]

- The definition of \( c_v \) and \( F_a \) are as given below.

\[ c_v = \frac{R}{\gamma - 1} \quad T_2' - T_1' = F_a (T_2 - T_1) \]

- Substituting, we have

\[ \Delta e = \left( \frac{R}{\gamma - 1} + \frac{R}{2} \right) (T_2 - T_1) F_a \]

\[ \Delta e = \frac{F_a R}{2} (T_2 - T_1) \left( \frac{\gamma + 1}{\gamma - 1} \right) \]
Vacuum

- The mass flux per unit time is given by
  \[ \frac{\dot{m}}{A} = \frac{\rho \bar{v}}{4} \]

- where,
  - \( \rho \) – Density, \( \bar{v} \) – Average velocity

- From Kinetic theory, average velocity is
  \[ \bar{v} = \left( \frac{8RT}{\pi} \right)^{0.5} \]

- Combining the above, together with equation of state, we have
  \[ \frac{\dot{m}}{A} = \frac{1}{4} \left( \frac{p}{RT} \right) \left( \frac{8RT}{\pi} \right)^{0.5} \]
  \[ \frac{\dot{m}}{A} = p \left( \frac{1}{2\pi RT} \right)^{0.5} \]
Vacuum

- The total energy transfer per unit area owing to the molecular conduction is as given below.

\[
\frac{\dot{Q}}{A} = \frac{\dot{m}}{A} (\Delta e) = p \left( \frac{1}{2\pi RT} \right)^{0.5}
\]

\[
\Delta e = \frac{F_a R}{2} \left( T_2 - T_1 \right) \left( \frac{\gamma + 1}{\gamma - 1} \right)
\]

\[
\frac{\dot{Q}}{A} = p \left( \frac{1}{2\pi RT} \right)^{0.5} \left( \frac{F_a R}{2} \left( T_2 - T_1 \right) \left( \frac{\gamma + 1}{\gamma - 1} \right) \right)
\]

\[
\frac{\dot{Q}}{A} = \left( \frac{\gamma + 1}{\gamma - 1} \right)^{0.5} \left( \frac{R}{8\pi T} \right)^{0.5} F_a p (T_2 - T_1)
\]

- T is the temperature of the pressure gauge measuring the gas pressure.
Vacuum

\[
\frac{\dot{Q}}{A} = \left( \frac{\gamma + 1}{\gamma - 1} \right) \left( \frac{R}{8\pi T} \right)^{0.5} F_a p(T_2 - T_1)
\]

- In the above equation, let us denote the term in the parenthesis by \( G \). We have,

\[
\dot{Q} = G \ p \ A \ (T_2 - T_1)
\]

- \( Q \) is valid only when the distance (\( L \)) between the plates is less than the mean free path (\( \lambda \)). Mathematically,

\[
L < \lambda = \frac{\mu}{p} \left( \frac{\pi RT}{2} \right)^{0.5}
\]
Vacuum

\[ \dot{Q} = G_0 p A (T_2 - T_1) \]

\[ \lambda = \frac{\mu}{p} \left( \frac{\pi RT}{2} \right)^{0.5} \]

- From the above two equations, it is clear that the
  - The free molecular regime can be achieved by achieving very good vacuum.

- The free molecular conduction heat transfer can be made negligible compared to other modes, by lowering the pressure, decreasing \( F_a \), decreasing \( (T_2 - T_1) \).
Evacuated Powder

• Gas conduction is the primary and the dominant mode of heat transfer in a gas filled powder and fibrous insulations.

• One of the obvious ways to reduce this heat transfer is to evacuate the powder and the fibrous insulations.

• Usually, the vacuum that is commonly maintained in these insulations is in the range of $10^3$ to $10^{-5}$ torr. 1 torr = 1 mm of Hg.
Evacuated Powder

- The adjacent figure shows the variation of $k_A$ with the residual gas pressure inside an evacuated powder insulation.

- $k_A$ is independent of residual gas pressures lying between atmospheric and 15 torr.
• With the lowering of pressure, 15 torr to $10^{-3}$ torr, $k_A$ becomes directly proportional to the pressure.

• It varies almost linearly on a logarithmic chart as shown.

• Here, the modes of heat transfer are due to radiation, solid conduction and free molecular conduction (dominant).
Evacuated Powder

- With the further lowering of pressure, below $10^{-3}$ torr, the variation of $k_A$ is almost null.

- The mode of heat transfer is primarily due to solid conduction and radiation.

- Evacuated powders are superior in performance than vacuum alone in $300-77$ K, as the radiation heat transfer is comparatively less.
At low pressures and temperatures, the solid conduction in evacuated powder dominates the radiant heat transfer.

Hence, it is more advantageous to use vacuum alone in 77 K to 4 K. From Fourier's Law, we have

\[ \dot{Q} = \frac{k_A A_m (T_h - T_c)}{\Delta x} \]
Evacuated Powder

\[ \dot{Q} = \frac{k_A A_m (T_h - T_c)}{\Delta x} \]

- where,
  - \( k_A \) = Apparent thermal conductivity
  - \( T_h - T_c \) = Temperature difference
  - \( \Delta x \) = Distance
  - \( A_m \) = Mean area of insulation. \( A_m \) for concentric cylinders and concentric spheres is as given below.

\[ A_{m,cyl} = \frac{A_2 - A_1}{\ln \frac{A_2}{A_1}} \]

\[ A_{m,sph} = \left( \frac{A_1 A_2}{2} \right)^{\frac{1}{2}} \]
Evacuated Powder

- The apparent thermal conductivity and density of few commonly used evacuated powder insulations are as shown.

- The residual gas pressure is less than $10^{-3}$ torr for temperatures between $77 \text{ K}$ to $300 \text{ K}$.

<table>
<thead>
<tr>
<th>Powder</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$k$ (mW/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine Pertile</td>
<td>180</td>
<td>0.95</td>
</tr>
<tr>
<td>Coarse Perlite</td>
<td>64</td>
<td>1.90</td>
</tr>
<tr>
<td>Lampblack</td>
<td>200</td>
<td>1.20</td>
</tr>
<tr>
<td>Fiberglass</td>
<td>50</td>
<td>1.70</td>
</tr>
</tbody>
</table>
Opacified Powder Insulation

• Radiation heat transfer still contributes to the heat in leak in $300 \, K$ to $77 \, K$ temperature range in case of evacuated powders.

• In the year 1960, Riede and Wang, Hunter et al. minimized this radiant heat transfer by addition of reflective flakes made of Al or Cu to the evacuated powder.

• These flakes act like radiant shields in the tiny heat transfer paths that are formed in the interstices of the evacuated powder.
The figure shows the variation of % opacifier with thermal conductivity for Cu – santocel and Al – santocel.

There exists an optimum operating point for each of these insulations.

It has been observed that, with these additions, $k_A$ can be reduced by 5 times.
- Cu flakes are more preferred as compared to Al flakes.
- The Al flakes have large heat of combustion.
- These together with O₂ can lead to accidents when used on LOX containers.
• Another disadvantage of this insulation is that the vibrations tend to pack the flakes together.

• This, not only increases the thermal conductivity but also short circuits the conduction heat transfer.
Opacified Powder Insulation

- The apparent thermal conductivity (mW/mK) and density (kg/m³) of few commonly used opacified powder insulations are as shown.

- The residual gas pressure is less than 10⁻³ torr for temperatures between 77 K to 300 K.

<table>
<thead>
<tr>
<th>Powder</th>
<th>ρ (kg/m³)</th>
<th>k (mW/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50/50 Cu – Santocel</td>
<td>180</td>
<td>0.33</td>
</tr>
<tr>
<td>40/60 Al – Santocel</td>
<td>160</td>
<td>0.35</td>
</tr>
<tr>
<td>50/50 Bronze – Santocel</td>
<td>179</td>
<td>0.58</td>
</tr>
<tr>
<td>Silica – Carbon</td>
<td>80</td>
<td>0.48</td>
</tr>
</tbody>
</table>
A spherical LN2 vessel (ε=0.8) is as shown. The inner and outer radii are 1.2m and 1.6m respectively. Compare and comment on the heat in leak for the following cases.

- Perlite (26 mW/mK)
- Less Vacuum (1.5mPa)
- Vacuum alone
- Vacuum + 10 shields (ε_s=0.05)
- Evacuated Fine Perlite (0.95 mW/mK)
- 50/50 Cu – Santocel (0.33 mW/mK)
## Tutorial

### Given

<table>
<thead>
<tr>
<th>Apparatus</th>
<th>Spherical vessel ((e=0.8))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working Fluid</td>
<td>Liquid Nitrogen</td>
</tr>
<tr>
<td>Temperature</td>
<td>77 K (inner), 300 K (outer)</td>
</tr>
</tbody>
</table>

### Calculate heat in leak

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Perlite (26 mW/mK)</td>
</tr>
<tr>
<td>2</td>
<td>Less Vacuum (1.5mPa)</td>
</tr>
<tr>
<td>3</td>
<td>Vacuum alone</td>
</tr>
<tr>
<td>4</td>
<td>Vacuum + 10 shields</td>
</tr>
<tr>
<td>5</td>
<td>Evacuated Fine Perlite (0.95 mW/mK)</td>
</tr>
<tr>
<td>6</td>
<td>50/50 Cu – Santocel (0.33 mW/mK)</td>
</tr>
</tbody>
</table>

- The shape factor between the two containers is assumed to be 1.
Tutorial

Perlite \((k_A = 26\text{mW/m-K})\)

- **Sphere** - \(R_1 = 1.6\text{m}, \ R_2 = 1.2\text{m}, \ k_A, \Delta T = (300 - 77) = 223.\)

\[
Q = \frac{4\pi k_A R_1 R_2 \Delta T}{(R_2 - R_1)}
\]

\[
Q = \frac{4\pi (26) (10^{-3}) (1.6)(1.2)(223)}{(1.6 - 1.2)}
\]

\[
Q = 349.7W
\]
Less Vacuum (1.5mPa)

- Sphere - \( R_1 = 1.6\text{m}, R_2 = 1.2\text{m}, \)
  \( e_1 = e_2 = 0.8, T_1 = 77\text{ K}, T_2 = 300\text{ K}. \)

- The net heat transfer is due to both radiation and residual gas conduction.

\[
F_e = \left( \frac{1}{e_1} + \left( \frac{A_1}{A_2} \right) \left( \frac{1}{e_2} - 1 \right) \right)^{-1}
\]

\[
F_e = \left( \frac{1}{0.8} + \left( \frac{1.2}{1.6} \right)^2 \left( \frac{1}{0.8} - 1 \right) \right)^{-1} = 0.72
\]
Tutorial

Less Vacuum (1.5mPa)

- **Sphere** - $R_1=1.6\text{m}$, $R_2=1.2\text{m}$, $e_1=e_2=0.8$, $T_1=77\text{ K}$, $T_2=300\text{ K}$.

\[
Q = F_e F_{1 \rightarrow 2} \sigma A_1 \left( T_2^4 - T_1^4 \right)
\]

\[
F_e = 0.72
\]

\[
Q = (0.72)(1)(5.67)(10^{-8})\pi(1.6^2)(300^4 - 77^4)
\]

\[
Q_r = 2648\text{W}
\]
Less Vacuum (1.5 mPa)

**Sphere** - $R_1=1.6 \text{ m}$, $R_2=1.2 \text{ m}$, $T_1=77 \text{ K}$, $T_2=300 \text{ K}$, $p=1.5 \text{ mPa}$.

\[
\lambda = \frac{\mu (\pi RT)}{p \left( \frac{2}{2} \right)}
\]

\[
\lambda = \frac{(18.47)(10^{-6})}{(1.5)(10^{-3})} \left( \frac{\pi (287.6)(300)}{2} \right)^{0.5} = 4.53
\]

- It is clear that the mean free path ($\lambda$) is greater than distance between the surfaces (0.4 m).
Tutorial

Less Vacuum (1.5 mPa)

- **Sphere** - $R_1 = 1.6$ m, $R_2 = 1.2$ m, $T_1 = 77$ K, $T_2 = 300$ K, $p = 1.5$ mPa.

\[ F_a = \left( \frac{1}{\alpha_1} + \left( \frac{A_1}{A_2} \right) \left( \frac{1}{\alpha_2} - 1 \right) \right)^{-1} \]

\[ F_e = \left( \frac{1}{1} + \left( \frac{1.2}{1.6} \right)^2 \left( \frac{1}{0.85} - 1 \right) \right)^{-1} = 0.91 \]

<table>
<thead>
<tr>
<th>T (K)</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.8-0.9</td>
</tr>
<tr>
<td>78</td>
<td>1.0</td>
</tr>
<tr>
<td>20</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Tutorial

Less Vacuum (1.5 mPa)

- **Sphere** - $R_1 = 1.6 \text{ m}$, $R_2 = 1.2 \text{ m}$, $T_1 = 77 \text{ K}$, $T_2 = 300 \text{ K}$, $p = 1.5 \text{ mPa}$.

\[
\dot{Q} = \left( \frac{\gamma + 1}{\gamma - 1} \right) \left( \frac{R}{8\pi T} \right)^{0.5} F_a p A (T_2 - T_1)
\]

\[
\dot{Q} = \left( \frac{1.4 + 1}{1.4 - 1} \right) \left( \frac{287.6}{8\pi (300)} \right)^{0.5} (0.91) (1.5) (10^{-3}) (300 - 77)
\]

\[
Q_{gc} = 0.356 W
\]
Vacuum alone

**Sphere** - $R_1 = 1.6 m$, $R_2 = 1.2 m$, $k_A$, $T_1 = 77 K$, $T_2 = 300 K$, $e_1$, $e_2 = 0.8$, $F_{1\rightarrow 2} = 1.$

\[ Q = F_e F_{1\rightarrow 2} \sigma A_1 \left( T_2^4 - T_1^4 \right) \]

\[ F_e = 0.72 \]

\[ Q = (0.667)(1)(5.67)(10^{-8})\pi(1.6^2)(300^4 - 77^4) \]

\[ Q = 2648 W \]

Vacuum + 10 shields

- $e_1$, $e_2 = 0.8$, $e_s = 0.05.$

\[ F_e = 0.003 \]

\[ Q = 11.02 W \]
Evacuated Fine Perlite ($k_A = 0.95 \text{mW/mK})

- **Sphere** - $R_1 = 1.6 \text{m}$, $R_2 = 1.2 \text{m}$, $k_A$, $\Delta T = (300-77) = 223$.

$$Q = \frac{4\pi k_A R_1 R_2 \Delta T}{(R_2 - R_1)}$$

$$Q = 12.7 \text{W}$$

50/50 Cu – Santocel ($k_A = 0.33 \text{mW/m-K}$)

- **Sphere** - $R_1 = 1.6 \text{m}$, $R_2 = 1.2 \text{m}$, $k_A$, $\Delta T = (300-77) = 223$.

$$Q = \frac{4\pi k_A R_1 R_2 \Delta T}{(R_2 - R_1)}$$

$$Q = 4.41 \text{W}$$
## Tutorial

### Heat in leak (Q)

<table>
<thead>
<tr>
<th>Material</th>
<th>Q (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perlite</td>
<td>349.7 W</td>
</tr>
<tr>
<td>Less Vacuum (1.5 mPa)</td>
<td>( Q_r = 2648 ) W, ( Q_{gc} = 0.356 ) W</td>
</tr>
<tr>
<td>Vacuum alone</td>
<td>2648 W</td>
</tr>
<tr>
<td>Vacuum + 10 shields</td>
<td>11.02 W</td>
</tr>
<tr>
<td>Evacuated Fine Perlite</td>
<td>12.7 W</td>
</tr>
<tr>
<td>50/50 Cu – Santocel</td>
<td>4.41 W</td>
</tr>
</tbody>
</table>
Summary

• In vacuum, the radiation is the dominant mode of heat transfer.

• Evacuated powders are superior in performance than vacuum alone in 300-77 K, as the radiation heat transfer is comparatively less.

• At low pressures and temperatures, the solid conduction in evacuated powder dominates the radiant heat transfer.

• In an opacified powder, the radiation heat transfer is minimized by addition of reflective flakes.
Thank You!