There are four tutorial problems in the forthcoming slides.

1. Thermal expansion/contraction – 1 tutorial.

2. Estimation of $C_v$ using the Debye Theory – 2 tutorials.

3. Thermal conductivity of materials – 1 tutorial.
Calculate the overlap length of a brazed butt joint formed by SS 304 ($L_0=1\text{m}$) and Copper ($L_0=0.5\text{m}$). It is desired that the minimum overlap should be greater than 5mm. The joint is subjected to a low temperature of 80 K. Use the following data for the calculations.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>SS $\Delta L/L_0 \cdot 10^5$</th>
<th>Copper $\Delta L/L_0 \cdot 10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 K</td>
<td>304</td>
<td>337</td>
</tr>
<tr>
<td>80 K</td>
<td>13</td>
<td>26</td>
</tr>
</tbody>
</table>

Overlap $\geq 5\text{mm}$

- This condition should be verified at 80 K.
**SS 304**
- Mean linear expansion in SS 304 butt

\[
\frac{\Delta L_{SS}}{L_0} = \left( \frac{L_{T1}}{L_0} - \frac{L_{T2}}{L_0} \right) \times 10^{-5}
\]

\[
\frac{\Delta L_{SS}}{L_0} = (304 - 13) \times 10^{-5}
\]

\[L_0 = 1m, \Delta L_{SS} = 2.91mm\]

**Cu**
- Mean linear expansion in Cu butt

\[
\frac{\Delta L_{Cu}}{L_0} = \left( \frac{L_{T1}}{L_0} - \frac{L_{T2}}{L_0} \right) \times 10^{-5}
\]

\[
\frac{\Delta L_{Cu}}{L_0} = (337 - 26) \times 10^{-5}
\]

\[L_0 = 1m, \Delta L_{Cu} = 3.11mm\]

\[L_0 = 0.5m, \Delta L_{Cu} = 1.55mm\]
Tutorial – 1

- The greater of the two expansions is $dL_{SS}$
- The safe Butt joint should be more than $dL_{SS} + 5 = 7.91$mm.

Overlap = 8.1mm (say)

- When this joint is cooled to 80 K, the butt width in Cu after shrinkage is 6.55mm. Similarly, the butt width in SS after shrinkage is 5.19mm.

- Hence, the overlap being more than 5mm is a good design.
Debye Theory

- The expression for $C_v$, given by Debye theory is

\[ C_v = 3R \left( \frac{T}{\Theta_D} \right)^3 D \left( \frac{T}{\Theta_D} \right) \]

- $\Theta_D$ is called as Debye Characteristic Temperature.

- At $(T > 2\Theta_D)$, $C_v$ approaches $3R$. This is called as Dulong and Petit Value.

- At $(T < \Theta_D/12)$, $C_v$ is given by following equation.

\[ C_v = \frac{12\pi^4 R}{5} \left( \frac{T}{\Theta_D} \right)^3 \]

- Also, $D(0)$ is given a constant value of $4\pi^4/5$. 
Specific Heat Curve

- The variation of $C_v/R$ with $T/\theta_D$ is as shown.
- $\theta_D$ for few materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\theta_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>390</td>
</tr>
<tr>
<td>Lead</td>
<td>86</td>
</tr>
<tr>
<td>Nickel</td>
<td>375</td>
</tr>
<tr>
<td>Copper</td>
<td>310</td>
</tr>
<tr>
<td>Silver</td>
<td>220</td>
</tr>
<tr>
<td>$\alpha$-Iron</td>
<td>430</td>
</tr>
<tr>
<td>Titanium</td>
<td>350</td>
</tr>
</tbody>
</table>
Tutorial – 2

Determine the lattice specific heat of copper at 100 K. Given that the molecular weight is 63.54 g/mol.

• Step 1:
• Calculation of $T/\theta_D$ ratio.

\[
T = 100 \text{ K} \\
\theta_D = 310 \text{ K}
\]

\[
\frac{T}{\theta_D} = \frac{100}{310} = 0.3225
\]

• The value of $T/\theta_D$ is greater than $1/12$ ($0.0833$).

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<tr>
<td>Titanium</td>
<td>350</td>
</tr>
</tbody>
</table>
• The value of $T/\theta_D = 0.3225$.

• From the graph, $C_v/R = 1.93$.

\[
R = \frac{8.314}{0.06354} = 130.85
\]

\[
C_v = 130.85 \times 1.93
\]

\[
= 252.534 \text{ J/kg-K}
\]
Tutorial – 3

Determine the lattice specific heat of Aluminum at 25 K. Given that the molecular weight is 27 g/mol.

• Step 1:

• Calculation of $T/\theta_D$ ratio.

\[
\frac{T}{\theta_D} = \frac{25}{390} = 0.0641
\]

• The value of $T/\theta_D$ is less than $1/12$ (0.0833).

<table>
<thead>
<tr>
<th>Material</th>
<th>$\theta_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
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</tr>
<tr>
<td>Lead</td>
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</table>
Tutorial – 3

- Since, the $T/\theta_D$ ratio is less than $1/12$, the equation to calculate the specific heat is as given below.

$$c_v = \frac{12\pi^4 R}{5} \left( \frac{T}{\theta_D} \right)^3$$

$$R = \frac{8.31434}{0.0270} = 307.9$$

$$c_v = \frac{233.78RT^3}{\theta_D^3}$$

$$= 18.958 \text{ J/kg-K}$$
Thermal Conductivity Integrals

• The Fourier’s Law of heat conduction is

\[ Q = -k(T)A(x) \frac{dT}{dx} \]

• To make calculations less difficult and to account for the variation of \( k \) with temperature, \( Q \) is expressed as

\[ Q = -G(\theta_2 - \theta_1) \]

• \( kdT \) is taken as an integral called as Thermal Conductivity Integral evaluated w.r.t a datum.

\[ \theta_1 = \int_{T_d}^{T_1} k(T) dT \]

• If \( A_{cs} \) is constant, \( G \) is defined as

\[ G = \frac{A_{cs}}{L} \]

For Example

\( T_d = 0 \) or 4.2

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Thermal Conductivity Integrals

The variation of $kdT$ for few of the commonly used materials is as shown.

In the calculations, the actual temperature distribution is not required, but only the end point temperatures.

\[
\int_{10}^{100} kdT = \int_{0}^{100} kdT - \int_{0}^{10} kdT
\]
Tutorial – 4

Determine the heat transferred in an copper slab of uniform cross section area 1cm$^2$ and length of 0.1m, when the end faces are maintained at 300 K and 80 K respectively. Compare the heat transferred by $k_{\text{avg}}$ and $kdT$ methods.

**Given**

- Area of cross section : $10^{-4}$ m$^2$
- Length of specimen: 0.1 m
- $T_1 = 300$ K
- $T_2 = 80$ K
**Tutorial – 4**

**k$_{\text{avg}}$ Method**

\[ Q = -k_{\text{avg}} A \frac{dT}{dx} \]

\[ Q = k_{\text{avg}} A \left( \frac{T_1 - T_2}{L} \right) \]

\[ Q = 57.75 \times 10^{-4} \left( \frac{300 - 80}{0.1} \right) \]

\[ Q = 18.958 \text{ W} \]

\[ k_{300} = 78.5 \text{ W/m K} \]

\[ k_{80} = 37.0 \text{ W/m K} \]

\[ k_{\text{avg}} = 57.75 \text{ W/m K} \]
kdT Method

\[ Q = -G(\theta_2 - \theta_1) \]

\[ \theta_1 = \int_{4.2}^{300} k(T) dT = 15000 \]

\[ \theta_2 = \int_{4.2}^{80} k(T) dT = 1600 \]

\[ Q = -\frac{10^{-4}}{0.1} (1600 - 15000) \]

\[ Q = 13.4 \text{ W} \]

Comparison

<table>
<thead>
<tr>
<th>( k_{avg} )</th>
<th>kdT</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.98</td>
<td>13.4</td>
</tr>
</tbody>
</table>

- \( K_{avg} \) is more than the kdT method.