Tutorial

• Consider a rectification column for $N_2$ and $O_2$ separation operating at 1 atm. Determine the number of theoretical plates required to yield 97% $N_2$ at top and 95% $O_2$ at bottom. Feed stream is 50% $N_2$ and 50% $O_2$. Molar fraction of liquid in feed stream is 0.7 mole liquid/mole mixture. The desired flow rate at the bottom product is 20 mole/sec and the heat removed in the condenser at top of the column is 500 kW.
### Tutorial

**Given**
- Working Pressure: 1 atm
- Mixture: $N_2 + O_2$
- Feed stream: 50% $N_2 + 50% O_2$
- Bottom flow rate: 20 mole/sec = $B$
- Feed liq.: 0.7 = $q$

### For above mixture
- Reqt. of $N_2$ (top): 97% = $x_D$
- Reqt. of $O_2$ (bottom): 95% = $x_B = 0.05$
- Total number of theoretical plates
Tutorial

- Mole balance

\[ F = B + D \]

\[ x_F F = x_B B + x_D D \]

- Mole balance for \( \text{N}_2 \)

<table>
<thead>
<tr>
<th>Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_F )</td>
<td>0.5</td>
</tr>
<tr>
<td>( x_B )</td>
<td>0.05</td>
</tr>
<tr>
<td>( x_D )</td>
<td>0.97</td>
</tr>
<tr>
<td>( B )</td>
<td>20</td>
</tr>
</tbody>
</table>

\[ F = 20 + D \]

\[ 0.5F = (0.05)(20) + (0.97)D \]

- Solving, we have
- \( F=39.14 \) mole/sec, \( D=19.14 \) mole/sec.
**Tutorial**

- **Enthalpy balance**

\[ \dot{Q}_B = \dot{Q}_D + h_D D + h_B B - h_F F \]

- **Fraction of stream in feed**

\[ q = \frac{H - h_F}{H - h} \]

- **Rearranging, we have**

\[ h_F = qh + (1 - q)H \]

- **For 50% N\textsubscript{2} + 50% O\textsubscript{2}**
  - \( h = 1084 \text{ J/mol} \)
  - \( H = 6992 \text{ J/mol} \)
Tutorial

\[ h_F = qh + (1 - q)H \]

**Data**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>H</td>
<td>6992 J/m</td>
</tr>
<tr>
<td>q</td>
<td>0.7</td>
</tr>
</tbody>
</table>

\[ h_F = (0.7)1084 + (1 - 0.7)6992 \]

\[ h_F = 2856.4 J/mol \]
\[ \dot{Q}_B = \dot{Q}_D + h_D D + h_B B - h_F F \]

**Data**

<table>
<thead>
<tr>
<th>Q_D</th>
<th>500 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>h_D</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>h_B</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>h_F</td>
<td>2856.4 J/m</td>
</tr>
</tbody>
</table>

\[ \dot{Q}_B = (500)10^3 + (1084)(19.14) + (1084)(20) - (2856.4)(39.14) \]

\[ \dot{Q}_B = 430.6 kW \]
Operating line for **Enriching Section**

\[
\frac{D}{V_n} = \frac{H_n - h_{n+1}}{\dot{Q}_D / D + h_D - h_{n+1}}
\]

**Data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_D)</td>
<td>500 kW</td>
</tr>
<tr>
<td>(H_n)</td>
<td>6992 J/m</td>
</tr>
<tr>
<td>(h_{n+1})</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>(h_D)</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>(D)</td>
<td>19.14 mol/s</td>
</tr>
</tbody>
</table>

**Calculation**

\[
\frac{D}{V_n} = 0.226
\]

\[
\frac{L_{n+1}}{V_n} = 1 - \frac{D}{V_n}
\]

\[
\frac{L_{n+1}}{V_n} = 1 - 0.226 = 0.773
\]

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- Operating line for **Enriching Section**

\[ y_n = \left( \frac{L_{n+1}}{V_n} \right) x_{n+1} + \left( \frac{D}{V_n} \right) x_D \]

\[ \frac{D}{V_n} = 0.226 \]

\[ \frac{L_{n+1}}{V_n} = 0.773 \]

\[
y_n = (0.773) x_{n+1} + (0.226)(0.97)
\]

\[
y_n = 0.773 x_{n+1} + 0.22
\]

**Data**

| \(x_D\) | 0.97 |
### Tutorial

- Operating line for **Stripping Section**

\[
\frac{B}{V_m} = \frac{H_m - h_{m+1}}{\dot{Q}_B - h_B + h_{m+1}}
\]

<table>
<thead>
<tr>
<th>Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_B)</td>
<td>430.6 kW</td>
</tr>
<tr>
<td>(H_m)</td>
<td>6992 J/m</td>
</tr>
<tr>
<td>(h_{m+1})</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>(h_B)</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>(B)</td>
<td>20 mol/s</td>
</tr>
</tbody>
</table>

\[
\frac{B}{V_m} = \frac{6992 - 1084}{(430.6)10^3} - \frac{1084 + 1084}{20} = 0.274
\]

\[
\frac{L_{m+1}}{V_m} = 1 + \frac{B}{V_m}
\]

\[
\frac{L_{m+1}}{V_m} = 1 + 0.274 = 1.274
\]

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- Operating line for **Stripping Section**

\[
y_m = \left( \frac{L_{m+1}}{V_m} \right) x_{m+1} - \left( \frac{B}{V_m} \right) x_B
\]

\[
\frac{B}{V_m} = 0.274
\]

\[
\frac{L_{m+1}}{V_m} = 1.274
\]

\[
y_m = (1.274) x_{m+1} - (0.274)(0.05)
\]

\[
y_m = 1.274 x_{m+1} - 0.013
\]

**Data**

| \(x_B\) | 0.05 |

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• Equation of Feed Line

\[ q = \frac{H - h_f}{H - h} \]

\[ y = \frac{q}{q - 1} x + \frac{x_F}{1 - q} \]

\[ y = \frac{0.7}{0.7 - 1} x + \frac{0.5}{1 - 0.7} \]

\[ y = -2.34x + 1.67 \]

<table>
<thead>
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<td>6992 J/m</td>
</tr>
<tr>
<td>h_f</td>
<td>2856.4 J/m</td>
</tr>
<tr>
<td>h</td>
<td>1084 J/m</td>
</tr>
<tr>
<td>x_F</td>
<td>0.5</td>
</tr>
</tbody>
</table>
• Summarizing, we have the following.

• OP line for enriching section:
  \[ y_n = 0.773x_{n+1} + 0.22 \]

• OP line for stripping section:
  \[ y_m = 1.274x_{m+1} - 0.013 \]

• q line:
  \[ y = -2.34x + 1.67 \]

• The stair casing procedure is shown on an excel sheet to have a better understanding of the method.
Tutorial

• From the excel sheet, it is clear that the total number of vertical lines are 9.

• Therefore, the total number of theoretical plates for this column can be tabulated as shown below.

<table>
<thead>
<tr>
<th>McCabe – Thiele Method</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Enriching Section</td>
<td>3 + 1 (Condenser)</td>
</tr>
<tr>
<td>Stripping Section</td>
<td>6 + 1 (Boiler)</td>
</tr>
</tbody>
</table>