PID Controller Design

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This Lecture Contains

➢ What is PID Control?

➢ Advantage of PID

➢ Advantage of PI and PD Controller

➢ Assignment
What is PID Control?

- Dynamic Systems are often controlled with the help of a three term compensator known as PID; P – stands for Proportional Control, I stands for Integral Control and D stands for Derivative Control.
- Consider a closed loop system with unity feedback as follows:

![PID Control Diagram](image)

- The plant $P$, is controlled by a control input $u(t)$, which can be expressed as follows:

$$u(t) = K_p e(t) + K_i \int e(t) \, dt + K_d \dot{e}(t)$$
Frequency Domain Representation of Controller

- The control input could be represented in frequency domain as follows:

\[ U(s) = [K_p + \frac{K_i}{s} + K_d s] E(s) \]

- The closed loop transfer function could be written as follows:

\[ \frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)} \]

- This tells us that the poles of the closed loop transfer functions are actually the zeros of \(1+C(s)P(s)\). By considering the three terms as three parameters, you can study the effect of changing each one of these parameters on the root locus.
Advantage of Different Parameters of PID Controller

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<tr>
<th>Parameters</th>
<th>Advantage</th>
<th>Limitation</th>
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<tr>
<td>K_p</td>
<td>Adjustment of Controller output</td>
<td>May cause instability</td>
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<tr>
<td>K_i</td>
<td>Produces zero steady state error</td>
<td>Slow dynamic Response and Instability</td>
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<tr>
<td>K_d</td>
<td>Provides rapid system response</td>
<td>Sensitive to Noise and non-zero offset</td>
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Application of PI Controller on a First Order System

- Consider a first order plant as follows:

\[ P(s) = \frac{K}{1 + \tau s} \]

- If we apply a PI controller then \( C(s) \) becomes:

\[ C(s) = K_p + \frac{K_i}{s} = K_p \frac{s + K_i}{s} \]

- The closed loop transfer function may be written as:

\[ \frac{Y(s)}{R(s)} = (KK_p) \frac{s + K_i / K_p}{\tau s^2 + (1 + K K_p)s + KK_i} \]
Numerical Simulation of the system

• Let us consider the first order system with $K=1$ and $\tau = 10$ s, hence the open loop system transfer function may be written as

$$T(s) = \frac{1}{1+10s}$$

• Let us look at the unit step response of this system – the controller has miserably failed to follow!
First Order System - Numerical Simulation

- Now let us consider a PI controller with the following parameters:

\[ K = 1 \]
\[ K_p = 1 \]
\[ K_i = 10 \]

- The new closed loop transfer function may be written as:

\[ T = \frac{s + 10}{10s^2 + 2s + 10} \]
Response of the new system

- The unit step response of the new system is shown below vis a vis the old system:
Application of PD Controller on a First Order System

• Consider a first order plant as follows:

\[ P(s) = \frac{K}{1 + \tau s} \]

• If we apply a PD controller then \( C(s) \) becomes:

\[ C(s) = K_p + K_d s = K_d \left( s + \frac{K_p}{K_d} \right) \]

• The closed loop transfer function may be written as:

\[ \frac{Y(s)}{R(s)} = \frac{KK_d s + KK_p}{(\tau + KK_d) s + (1 + K K_p)} \]
A numerical example

Let us consider a numerical example where $K=10$ and $\tau = 0.1$. The response of the system without any compensator is as follows: Note that it took about 0.05 seconds for the system to reach the steady state which is 0.9.
A Modified Response

- If you now add a compensator with $K_p = 9$ and $K_d = 1$, you can see the change in step response as follows. You may observe that the same value (0.9) is obtained in less than 0.2 seconds.
Assignment

Consider a second order system with natural frequency 10 Hz and Damping coefficient = 0.1. Find out a PID controller which will improve the steady state performance ten times and also improve the peak-time 5 times.

Hints: Consider the standard second order models discussed earlier and cascade it with a PID controller as shown in this lecture. Take trial values of $K_p$, $K_i$, and $K_d$ and find out the response – continue till the performance is satisfactory.
Special References for this lecture

- **Feedback Control of Dynamic Systems**, Frankline, Powell and Emami, Pearson
- *Control Systems Engineering* – Norman S Nise, John Wiley & Sons
- *Design of Feedback Control Systems* – Stefani, Shahian, Savant, Hostetter

Oxford